Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 923 (2017) 168–178
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Ratio of critical quantities related to Hawking temperature–entanglement entropy criticality

Jie-Xiong Mo *, Gu-Qiang Li

Institute of Theoretical Physics, Lingnan Normal University, Zhanjiang, 524048, Guangdong, China

Received 25 May 2017; received in revised form 30 July 2017; accepted 2 August 2017
Available online 8 August 2017
Editor: Stephan Stieberger

Abstract

We revisit the Hawking temperature–entanglement entropy criticality of the d-dimensional charged AdS black hole with our attention concentrated on the ratio $\frac{T_c \delta S_{Ec}}{Q_c}$. Comparing the results of this paper with those of the ratio $\frac{T_c S_c}{Q_c}$, one can find both the similarities and differences. These two ratios are independent of the characteristic length scale l and dependent on the dimension d. These similarities further enhance the relation between the entanglement entropy and the Bekenstein–Hawking entropy. However, the ratio $\frac{T_c \delta S_{Ec}}{Q_c}$ also relies on the size of the spherical entangling region. Moreover, these two ratios take different values even under the same choices of parameters. The differences between these two ratios can be attributed to the peculiar property of the entanglement entropy since the research in this paper is far from the regime where the behavior of the entanglement entropy is dominated by the thermal entropy.

1. Introduction

Entanglement entropy has received considerable attention these years for its important position in the AdS/CFT correspondence and widespread application in probing various physical phenomena. The Ryu–Takayanagi formula [1–3] shares striking similarity with the Bekenstein–Hawking entropy, implying that there may exist close relation between the entanglement entropy

* Corresponding author.
E-mail addresses: mojiexiong@gmail.com (J.-X. Mo), zsgqli@hotmail.com (G.-Q. Li).

http://dx.doi.org/10.1016/j.nuclphysb.2017.08.001

and black hole entropy. This relation sheds light on some deep physics. It was proposed that the entanglement entropy is the origin of black hole entropy [4–6]. Recently, Johnson [7] further enhanced this relation by disclosing intriguing phase structures of entanglement entropy. The isocharges in the entanglement entropy–temperature plane exhibit similarly to those of the thermal entropy–temperature plane. Moreover, it was shown that they share the same critical temperature and critical exponents [7]. Soon afterwards, the equal area law was proved to hold for the entanglement entropy–temperature plane [8], just as it holds for T–S (here, S denotes the thermal entropy of the black hole.) curve [9]. Ever since the pioneer work [7] of Johnson, the rich phase structures of various black holes have been investigated from the perspective of entanglement entropy [10–32].

In our recent paper [33], we concentrate on the ratios of critical physical quantities related to three different kinds of criticality of d-dimensional charged AdS black holes. Namely, T–S criticality, Q–Φ criticality and P–V criticality. In all these cases, we showed that there exist universal ratios that do not depend on the parameters. We also showed that the value of $\frac{T_cS_c}{Q_c}$ for T–S criticality differs from that of $\frac{P_cv_c}{T_c}$ for P–V criticality. Probing the universal ratios of critical quantities is of great physical significance. Disclosing the deep physics behind the phenomena of universal ratios will help draw a unified picture of black hole thermodynamics.

Considering the close relation between the entanglement entropy and the Bekenstein–Hawking entropy (as stated in the first paragraph), there may exist universal ratio of critical quantities related to the Hawking temperature–entanglement entropy criticality. This issue has not been covered in literature yet to the best of our knowledge. And investigating this ratio will be the target of this paper. We are about to resolve this issue within the framework of d-dimensional charged AdS black hole spacetime. The motivations are as follows. Firstly, we are curious about whether the analogous ratio of critical quantities related to the Hawking temperature–entanglement entropy criticality is also universal. In other words, does it depend on the parameters? Probing the universal ratios of critical quantities of charged AdS black holes has its own right. The analogy between charged AdS black holes and van der Waals liquid–gas system has gained extensive attention ever since the famous work [34,35]. The ratio $\frac{P_cv_c}{kT_c}$ is a universal number for all van der Waals fluids in classical thermodynamics, motivating us to searching for the universal ratios for charged AdS black holes. Secondly, we are interested in both the similarities and differences (if any) between the ratio for the Hawking temperature–entanglement entropy criticality and that for the Hawking temperature-thermal entropy criticality. On the one hand, the similarities will help further understand the close relation between the entanglement entropy and the Bekenstein–Hawking entropy. On the other hand, the differences may shed light on some yet unknown physics.

The organization of this paper is as follows. Sec. 2 devotes to a short review of three different kinds of criticality of d-dimensional charged AdS black holes. In Sec. 3, we will revisit the Hawking temperature–entanglement entropy criticality of the d-dimensional charged AdS black hole and investigate the analogous ratio of critical quantities for various cases where different parameters are chosen as the variable respectively. In the end, Sec. 4 devotes to conclusions.

2. A short review of criticality of charged AdS black holes

The metric of the d-dimensional ($d > 3$) charged AdS black hole reads

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \tag{1}$$