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Bubble nucleation and growth in very strong cosmological phase transitions

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Abstract

Strongly first-order phase transitions, i.e., those with a large order parameter, are characterized by a considerable supercooling and high velocities of phase transition fronts. A very strong phase transition may have important cosmological consequences due to the departures from equilibrium caused in the plasma. In general, there is a limit to the strength, since the metastability of the old phase may prevent the transition to complete. Near this limit, the bubble nucleation rate achieves a maximum and thus departs from the widely assumed behavior in which it grows exponentially with time. We study the dynamics of this kind of phase transitions. We show that in some cases a gaussian approximation for the nucleation rate is more suitable, and in such a case we solve analytically the evolution of the phase transition. We compare the gaussian and exponential approximations with realistic cases and we determine their ranges of validity. We also discuss the implications for cosmic remnants such as gravitational waves.

1. Introduction

A cosmological first-order phase transition may have several observable consequences, such as the generation of the baryon asymmetry of the universe (for a review, see [1]), the formation

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of topological defects [2], or the production of gravitational waves (see [3] for a review). The estimation of these relics involves computing the development of the phase transition in order to determine quantities such as the bubble wall velocity, the number density of nucleated bubbles, and the distribution of bubble sizes. Depending on the computational method, several approximations are usually necessary. In particular, it is customary to consider a constant wall velocity and specific analytical forms for the nucleation rate as a function of time. Among the latter, the most common assumptions are an exponentially growing rate, a constant rate, or just a simultaneous nucleation. The validity of these approximations depends on the strength of the phase transition.

For a first-order phase transition, the free energy density $\mathcal{F}$ is a function of an order parameter ϕ and has two minima separated by a barrier. The stable phase corresponds to the absolute minimum, while the metastable phase corresponds to a local minimum. For concreteness, we shall consider the case in which at high temperature T we have a stable minimum $\phi_+ = 0$, corresponding to a symmetric phase. At low temperatures, this minimum becomes metastable, while a broken-symmetry minimum ϕ_- becomes the stable one (we shall use the subscripts $\pm$ for quantities corresponding to each of these phases). At the critical temperature T_c the two minima of $\mathcal{F}(\phi, T)$ are degenerate. Several quantities, such as the energy density, are different in each phase. Therefore, if we assume that the phase transition takes place at $T = T_c$, these quantities are discontinuous functions of T. The discontinuities depend on the jump of ϕ. Thus, the value of ϕ_-/T at $T = T_c$ can be regarded as an order parameter. The phase transition is usually said to be weakly first order if $\phi_-/T \ll 1$ and strongly first order if $\phi_-/T \gtrsim 1$. We shall be mainly interested in the case of very strong phase transitions, for which $\phi_-/T \gg 1$.

At $T = T_c$ or higher, the bubble nucleation rate per unit volume, Γ, vanishes. Therefore, the phase transition does not occur exactly at $T = T_c$. Below T_c, the nucleation rate grows continuously from $\Gamma = 0$, and may reach, in principle, a value $\Gamma \sim T_c^4 \sim v^4$, where v is the energy scale characterizing the phase transition. In many cases there is a temperature $T_0 < T_c$ at which the barrier separating the two minima disappears and the local minimum $\phi_+ = 0$ becomes a maximum (see [4] for a review). At this temperature, the phase transition would proceed through spinodal decomposition rather than by bubble nucleation. Nevertheless, before reaching $T = T_0$ we would have $\Gamma \sim v^4$, which is extremely large. Indeed, the development of the phase transition depends crucially on the relation between Γ and the expansion rate H [6], since the number of bubbles N nucleated in a causal volume $V \sim H^{-3}$ in a cosmological time $t \sim H^{-1}$ will be $N \sim \Gamma H^{-4}$. While Γ varies significantly, H is roughly given by $H \sim T^2/M_P \sim v^2/M_P$, where M_p is the Planck mass. Thus, for T close enough to T_c we will have $\Gamma \ll H^4$ and the nucleation of bubbles will be too slow, while for $\Gamma \sim v^4$ we have $\Gamma H^{-4} \sim (M_P/v)^4 \gg 1$, since in most cases v is several orders of magnitude below M_P. Therefore, the phase transition will generally occur at an intermediate temperature between T_c and T_0, such that $\Gamma H^{-4} \sim 1$, and, due to the rapid growth of Γ, it will end in a time $\Delta t \ll t$.

For a weakly first-order phase transition, the barrier between minima is relatively small and the temperature T_0 is very close to T_c. On the other hand, for stronger phase transitions we have a larger barrier which persists at smaller temperatures or even at $T = 0$. In such a case there is no temperature T_0 (see [5] for examples of such models), and the phase transition can last longer.

Since in many cases the duration of the phase transition is short, the nucleation rate is sometimes approximated by a constant. This is convenient, e.g., for involved computations such as numerical simulations, in which sometimes it is even necessary to nucleate all the bubbles simultaneously. However, it is the variation of Γ with time what actually determines the dynamics of the transition. In general, the nucleation rate is of the form $\Gamma = A \exp(-S)$, where the quantities