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Exact solution for the inhomogeneous Dicke model in the canonical ensemble: Thermodynamical limit and finite-size corrections

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Abstract

We consider an exactly solvable inhomogeneous Dicke model which describes an interaction between a disordered ensemble of two-level systems with single mode boson field. The existing method for evaluation of Richardson–Gaudin equations in the thermodynamical limit is extended to the case of Bethe equations in Dicke model. Using this extension, we present expressions both for the ground state and lowest excited states energies as well as leading-order finite-size corrections to these quantities for an arbitrary distribution of individual spin energies. We then evaluate these quantities for an equally-spaced distribution (constant density of states). In particular, we study evolution of the spectral gap and other related quantities. We also reveal regions on the phase diagram, where finite-size corrections are of particular importance.

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1. Introduction

Recent progress in engineering of artificial quantum systems for information technologies renewed an interest to Dicke (Tavis–Cummings) model [1] and its exact solution, already well known for a long time [2–7]. Dicke model describes an interaction between a collection of two-level systems and single mode radiation field, while physical realizations range from superconducting qubits coupled to microwave resonators to polaritons in quantum wells, see e.g. Refs. [8,9] and references therein; furthermore, it can be also applied to Fermi–Bose condensates near the Feshbach resonance [10].

The characteristic feature of macroscopic artificial quantum systems such as superconducting qubits is a disorder in excitation frequencies and inhomogeneous broadening of the density of states. This feature is due to fundamental mechanisms: for example, an excitation energy of flux qubits depends exponentially on Josephson energies [11], which makes it extremely sensitive to characteristics of nanometer-scale Josephson junctions. Inhomogeneous broadening appears even in the case of microscopic two-level systems, such as NV-centers, where it is induced by spatial fluctuations of background magnetic moments [12]. In the case of NV-centers, the density of states is characterized by the q-Gaussian distribution [13]. Moreover, there are prospect to utilize the broadening for the construction of a multimodal quantum memory [14]. It is also possible to engineer a density of states profile by using, e.g., a so-called spectral hole burning technique which allows to perform a significant optimization of various characteristics of spin-photon hybrid systems [13].

Inhomogeneous Dicke model, which explicitly takes into account a disorder in excitation energies, has been studied in Ref. [15] using a mean-field treatment within functional-integral representation of the partition function. This study revealed an existence of a rather rich phase diagram. The interaction between boson and spin subsystems gives rise to a finite gap in the energy spectrum between the first excited state and the ground state. It has an apparent similarity with the superconducting gap in the Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity. The mean-field approximation for the inhomogeneous Dicke model also becomes exact in the thermodynamical limit, as for the BCS pairing Hamiltonian. However, this approximation is expected to fail in the mesoscopic regime, which seems to be more relevant for near-future technological applications with macroscopic artificial 'atoms', such as superconducting qubits. Indeed, structures, which consist of tens or hundreds of superconducting qubits and show signatures of global coherence, have been successfully fabricated and explored very recently [8,16]. Such structures are often refereed to as superconducting metamaterials.

The mesoscopic regime of the Dicke model as well as the emergence of the macroscopic limit can be properly described only using approaches based on a canonical ensemble, which takes into account that the 'particle' number is fixed. This circumstance makes it difficult to apply standard mean-field methods. By 'particle' number one should understand the total number of bosons and excited two-level systems (spins), so they can be referred to as pseudo-particles. The limiting validity of grand canonical description is well known in the case of pairing correlations in ultrasmall metallic systems at low temperatures and in nuclei, for which usual mean-field approximation can give results inadequate even on a qualitative level [17]. For example, it predicts vanishing of superconducting correlations below certain mean interlevel distance, while more advanced approaches show that they do not disappear. One of such approaches is to turn to Richardson–Gaudin solution of BCS pairing Hamiltonian [2,18] via Bethe ansatz technique, which was utilized to evaluate various characteristics along the crossover from few-particle systems to the macroscopic regime, see, e.g., Refs. [19,20]. In particular, finite-size corrections can