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Emergent fuzzy geometry and fuzzy physics in four dimensions

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Abstract

A detailed Monte Carlo calculation of the phase diagram of bosonic mass-deformed IKKT Yang–Mills matrix models in three and six dimensions with quartic mass deformations is given. Background emergent fuzzy geometries in two and four dimensions are observed with a fluctuation given by a noncommutative $U(1)$ gauge theory very weakly coupled to normal scalar fields. The geometry, which is determined dynamically, is given by the fuzzy spheres $\mathbf{S}^2_N$ and $\mathbf{S}^2_N \times \mathbf{S}^2_N$ respectively. The three and six matrix models are effectively in the same universality class. For example, in two dimensions the geometry is completely stable, whereas in four dimensions the geometry is stable only in the limit $M \longrightarrow \infty$, where M is the mass of the normal fluctuations. The behaviors of the eigenvalue distribution in the two theories are also different. We also sketch how we can obtain a stable fuzzy four-sphere $\mathbf{S}^2_N \times \mathbf{S}^2_N$ in the large N limit for all values of M as well as models of topology change in which the transition between spheres of different dimensions is observed. The stable fuzzy spheres in two and four dimensions act precisely as regulators which is the original goal of fuzzy geometry and fuzzy physics. Fuzzy physics and fuzzy field theory on these spaces are briefly discussed.

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1. Introduction

A commutative/noncommutative space in Connes' approach to geometry is given in terms of a spectral triple $(\mathcal{A}, \Delta, \mathcal{H})$ rather than in terms of a set of points [1]. $\mathcal{A}$ is the algebra of functions or bounded operators on the space, Δ is the Laplace operator or, in the case of spinors, the Dirac operator, and $\mathcal{H}$ is the Hilbert space on which the algebra of bounded operators and the differential operator Δ are represented.

In the IKKT model the geometry is in a precise sense emergent. The algebra $\mathcal{A}$ is given, in the large N limit, by Hermitian matrices with smooth eigenvalue distribution and bounded square trace [2]. The Laplacian/Dirac operator is given in terms of the background solution while the Hilbert space $\mathcal{H}$ is given by the adjoint representation of the gauge group $U(N)$.

In this article we will study IKKT Yang–Mills matrix models with quartic mass deformations in three and six dimensions with $SO(3)$ and $SO(3) \times SO(3)$ symmetries which will lead naturally to the fuzzy two-sphere $\mathbf{S}^2_N$ and to the fuzzy four-sphere $\mathbf{S}^2_N \times \mathbf{S}^2_N$ respectively.

Noncommutative gauge theory on the fuzzy two-sphere [3,4] was introduced in [10]. It was derived as the low energy dynamics of open strings moving in a background magnetic field with S^3 metric in [9]. This theory consists of the Yang–Mills term which can be obtained from the reduction to zero dimensions of ordinary $U(N)$ Yang–Mills theory in 3 dimensions and a Chern–Simons term due to Myers effect [18]. The model was studied perturbatively in [8] and [12] and nonperturbatively in [11]. This model contains beside the usual two-dimensional gauge field a scalar fluctuation normal to the sphere. In [7] a generalized model was proposed and studied in which this normal scalar field was suppressed by giving it a potential with very large mass. This was studied further in [5,6] where the instability of the sphere was interpreted along the lines of an emergent geometry phenomena.

In [13] an elegant random matrix model with a single matrix was shown to be equivalent to a gauge theory on the fuzzy sphere with a very particular form of the potential which in the large N limit leads to a decoupled normal scalar fluctuation. In [14–17] an alternative model of gauge theory on the fuzzy sphere was proposed in which field configurations live in the Grassmannian manifold $U(2N)/(U(N+1) \times U(N-1))$. In [14,15] this model was shown to possess the same partition function as the commutative model via the application of the powerful localization techniques.

Noncommutative gauge theory on the fuzzy four-sphere $\mathbf{S}^2_N \times \mathbf{S}^2_N$ which is given by a six matrix model with global $SO(3) \times SO(3)$ symmetry containing at most quartic powers of the matrices was proposed in [22]. The value $M = 1/2$ of the mass deformation parameter corresponds to the model studied [21] which can also be shown to correspond to a random matrix model with two matrices. This theory involves two normal scalar fields plus a four-dimensional gauge field. Again the mass deformation parameter M is essentially the mass of these normal fluctuations and thus for large M these scalar fields become weakly coupled. In [42] an interpretation of these normal scalar fields as dark energy as dark energy is put forward.

Before we give a summary of the main results reported in this article we will first summarize the main models studied here and their normalization. The most general mass-deformed IKKT Yang–Mills matrix model in three dimensions up to quartic power in the gauge field X_a is given by [5]

$$S[X] = NTr\left[-\frac{1}{4}[X_a, X_b]^2 + \frac{2i\alpha}{3}\epsilon_{abc}X_aX_bX_c + M(X_a^2)^2 + \beta X_a^2\right].$$

The Steinacker model [13] corresponds to $M = 1/2$ and