Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 913 (2016) 964–986

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Analytic representations of Yang–Mills amplitudes

N.E.J. Bjerrum-Bohr [a], Jacob L. Bourjaily [a,*], Poul H. Damgaard [a], Bo Feng [b]

[a] *Niels Bohr International Academy and Discovery Center, The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

[b] *Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou City, 310027, People's Republic of China*

Received 29 September 2016; accepted 17 October 2016

Available online 24 October 2016

Editor: Stephan Stieberger

Abstract

Scattering amplitudes in Yang–Mills theory can be represented in the formalism of Cachazo, He and Yuan (CHY) as integrals over an auxiliary projective space—fully localized on the support of the scattering equations. Because solving the scattering equations is difficult and summing over the solutions algebraically complex, a method of directly integrating the terms that appear in this representation has long been sought. We solve this important open problem by first rewriting the terms in a manifestly Möbius-invariant form and then using monodromy relations (inspired by analogy to string theory) to decompose terms into those for which combinatorial rules of integration are known. The result is the foundations of a systematic procedure to obtain analytic, covariant forms of Yang–Mills tree-amplitudes for any number of external legs and in any number of dimensions. As examples, we provide compact analytic expressions for amplitudes involving up to six gluons of arbitrary helicities.

1. Introduction

One of the most fundamental quantities in theoretical particle physics is the scattering amplitude for n gauge bosons. Although so essential, it is remarkable that for a long time explicit expressions for covariant d-dimensional scattering amplitudes of n massless gauge bosons of

* Corresponding author.
E-mail address: bourjaily@nbi.ku.dk (J.L. Bourjaily).

http://dx.doi.org/10.1016/j.nuclphysb.2016.10.012

arbitrary helicities were most easily obtained from the field theory limit of string theory (see, e.g., [1] for a review). Conventional d-dimensional Feynman diagram techniques are simply way too cumbersome above a small number of external legs. The highly efficient BCFW on-shell recursion relations [2,3] provide a practical solution, but it would still be worthwhile to explore alternate approaches.

In the scattering equation formalism of CHY, [4–6], represents a completely new step towards obtaining such compact covariant expressions for amplitudes. Expressed in terms of a (reduced) $2n \times 2n$ Pfaffian, the n-point S-matrix element is given by a $(n-3)$-dimensional integral which *fully localizes* on the set of solutions to so-called scattering equations. A proof of the validity of this remarkable formula for any n has been given in ref. [7] and it has also been derived from the viewpoint of the ambitwistor string [8–10]. Thus, no integrations are really required to find the n-point covariant scattering amplitude, only a sum over solutions to a set of algebraic equations. The downside of this is that the sum scales with n as $(n-3)!$ and finding the full set of solutions becomes difficult already at rather low values n. Progress has been made from a variety of different directions [11–13].[1]

Recently, a simple set of analytic integration rules were derived. They circumvent the problem of summing over $(n-3)!$ solutions and provides the result of that sum based on a simple combinatorial algorithm, [15–17]. However, some of the integrals needed in order to obtain explicit expressions for covariant gauge boson amplitudes were not immediately in a form where these simple integration rules were applicable. Rather, one would first have to resort to a not entirely systematic use of integration-by-parts identities. This makes it hard to provide general and simple rules for deriving any n-point gauge boson scattering amplitude using this formalism.

Very recently, the issue of integration rules for more general CHY integrands has been considered from two independent directions [18,19]. The monodromy relations solve such problems by shifting the integration contours appropriately. That way we rewrite all integrands in terms of pieces that all have $\alpha' \to 0$ limits without further analytic continuation. Other prescriptions with less compact integrands (e.g., rewritten through also integration by parts identities) can indeed be verified to be free of such terms. However, such prescriptions appear very hard to systematize. In this paper, we shall present a different and fully systematic solution to the problem—applicable at least to the case of integrands appearing in the CHY representation of Yang–Mills amplitudes. Interestingly, our method uses the idea of monodromy as it is applied in string theory [20,21]. This is perhaps puzzling on two counts. First, monodromy relations in string theory *a priori* only provide non-trivial relations between *full amplitudes*: by a sequence of contour shifts, and upon taking first real and then imaginary parts [20], one derives KK amplitude relations [22] and BCJ amplitude relations [23], respectively. Second, because the CHY construction is based on entirely different integrations on a set of δ-function constraints, it may not seem *a priori* obvious why monodromy considerations can apply to that formalism.

To understand the first issue, one should realize that monodromy in string theory is far more general than as applied to a full amplitude: it can also be applied to individual terms in the string theory integrand. To understand the second issue, one needs to know the intimate relationship between string theory integrals and CHY integrals, as explained in ref. [24] (see also section 3 of ref. [15]). The latter connection allows us to import monodromy relations of string theory in the $\alpha' \to 0$ limit into CHY integrands. In this way we establish a broad class of general relations sat-

[1] We are also aware of another approach to analytic integration—very different than what is described here—that should work for arbitrary CHY/string-theory integrands, [14].