Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 913 (2016) 987–1000
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

On $SL(2, R)$ symmetry in nonlinear electrodynamics theories

Komeil Babaei Velni [a,*], H. Babaei-Aghbolagh [b]

[a] *Department of Physics, Facualty of Science, University of Guilan, Rasht, Iran*
[b] *Department of Physics, Ferdowsi University of Mashhad, Mashhad, Iran*

Received 27 August 2016; received in revised form 25 October 2016; accepted 29 October 2016

Available online 5 November 2016

Editor: Stephan Stieberger

Abstract

Recently, it has been observed that the Noether–Gaillard–Zumino (NGZ) identity holds order by order in α' expansion in nonlinear electrodynamics theories as Born–Infeld (BI) and Bossard–Nicolai (BN). The nonlinear electrodynamics theory that couples to an axion field is invariant under the $SL(2, R)$ duality in all orders of α' expansion in the Einstein frame. In this paper we show that there are the $SL(2, R)$ invariant forms of the energy momentum tensors of axion-nonlinear electrodynamics theories in the Einstein frame. These $SL(2, R)$ invariant structures appear in the energy momentum tensors of BI and BN theories at all orders of α' expansion. The $SL(2, R)$ symmetry appears in the BI and BN Lagrangians as a multiplication of Maxwell Lagrangian and a series of $SL(2, R)$ invariant structures.

1. Introduction

Duality transformations in nonlinear electrodynamics has been studied in [1]. Classical electromagnetism is the most familiar duality-invariant theory. The Maxwell's Hamiltonian and the equations of motion are invariant under rotations. Note that the Lagrangian, however, is not

* Corresponding author.
E-mail addresses: komeilvelni@gmail.com (K. Babaei Velni), hossein.babaei66@gmail.com (H. Babaei-Aghbolagh).

http://dx.doi.org/10.1016/j.nuclphysb.2016.10.020
0550-3213/

invariant. This brings that nonlinear deformations of Lagrangian will require modifications which are also non-invariant.

Commonly, duality transformations may be found out in the path integral as a Legendre transform. Given some Lagrangian $L(F)$ depending only on the field strength of a vector field, one can construct [2]

$$\mathcal{L}(\tilde{F}, G) = \mathcal{L}(F) - \frac{1}{2}\epsilon^{abcd} F_{ab}\partial_c \tilde{A}_d \tag{1}$$

in which F is treated as a fundamental field. The classical equations of motion for F require that $G_{ab} = \partial_a \tilde{A}_b - \partial_b \tilde{A}_a$ is related to F by

$$G_{ab} = -2\frac{\partial \mathcal{L}(F)}{\partial F^{ab}} \tag{2}$$

through $G_{ab} = -\frac{1}{2}\epsilon_{abcd}\tilde{G}^{cd}$ and $\tilde{G}^{ab} = \frac{1}{2}\epsilon^{abcd}G_{cd}$.

Consistency of the duality constraint can be expressed as a requirement in which the Lagrangian must transform under duality in a particular way, defined by the Noether–Gaillard–Zumino (NGZ) identity [3].

We consider $\mathcal{L}_{inv}$ that is invariant under the following transformation:

$$\delta \begin{pmatrix} \tilde{F} \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \tilde{F} \\ G \end{pmatrix} \tag{3}$$

where $A^T = -D$, $B^T = B$ and $C^T = C$ are the infinitesimal parameters of the transformations [2]. By applying the duality symmetry, the NGZ identity could be written as following [4]:

$$\mathcal{L} = \mathcal{L}_{inv} - \frac{1}{4}G_{ab}F^{ab}. \tag{4}$$

When the theory only has linear duality (e.g. only F^2 terms in the action) $\delta L_{inv}/\delta F$ vanishes. So it could be found that any higher order dependence (F^4, $F^6, \ldots$) must be part of $\mathcal{L}_{inv}$. The NGZ identity, along with (2) can be solved to find $G(F)$ and various Lagrangians which provide a duality symmetry between equations of motion and Bianchi identities. We will discuss two cases of nonlinear deformations of the Maxwell theory that depend only on F's without derivatives.

The NGZ identity has a significant consequence for the duality rotations properties of the energy-momentum tensor. The energy-momentum tensor, which can be obtained as the variational derivative of the Lagrangian with respect to the gravitational field, is invariant under duality transformation [5]. It has been found as [6]

$$T_{ab} = g_{ab}\mathcal{L} + G_a{}^c F_{bc} \tag{5}$$

Considering this relation and (4), it could be found that $\mathcal{L}_{inv} = 1/4 T_a{}^a$.

The Born–Infeld Lagrangian density can be written intelligently in term of the square root of a determinant [7]:

$$\mathcal{L}_{BI} = \sqrt{-\det(\eta_{ab} + F_{ab})} - 1 \tag{6}$$

where the fundamental $(scale)^2$ ($= T^{-1} = 2\pi\alpha'$ in the string theory context) has been set equal to 1, has many considerable aspects, including electro-magnetic duality symmetry [8]. Since in four dimensions:

$$-\det(\eta_{ab} + F_{ab}) = \frac{1}{2}F_{ab}F^{ab} - \frac{1}{16}(F_{ab}\tilde{F}^{ab})^2, \qquad \tilde{F}^{ab} = \frac{1}{2}\epsilon^{abcd}F_{cd}, \tag{7}$$