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Relativistic causality and position space renormalization

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To the memory of Raymond Stora

Abstract

The paper gives a historical survey of the causal position space renormalization with a special attention to the role of Raymond Stora in the development of this subject. Renormalization is reduced to subtracting the pole term in analytically regularized primitively divergent Feynman amplitudes. The identification of residues with "quantum periods" and their relation to recent developments in number theory are emphasized. We demonstrate the possibility of integration over internal vertices (that requires control over the infrared behavior) in the case of the massless φ^4 theory and display the dilation and the conformal anomaly.

1. Introduction

As Raymond Stora had written[2] in his inimitable ironic style, he had *contributed to the "useful physics"* (in his work with P. Moussa on angular distributions in 2-particle reactions) *as well as to the "useless" quantum field theory (QFT), including the analysis of analytic properties of scattering amplitudes which follow from the causality principle* – in joint work with Bros, Epstein, Glaser, Messiah (see, e.g., [11]). Not surprisingly, our discussions at CERN were devoted to the "useless" part.

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[2] I thank Paul Sorba for providing me with Stora's "bio0908" for the French Academy.

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Perturbative ultraviolet renormalization in QFT was originally worked out for momentum space integrals beginning with a high energy cutoff. But a causal position space approach has also been developed concurrently by Ernst Stueckelberg, a Swiss student of Sommerfeld, starting in the early forties [19] (after a 1938 paper in German, anticipating the abelian Higgs–Kibble model, he switched to French – see [31–34]). This was taken up by a (French reading) mathematician, N.N. Bogolubov [3], who set himself to master QFT (while mobilized to work – with many others – on the Russian atomic project). The Russian work on renormalization (referred to in the book [4] – see, in particular, [29]), perfected by Hepp [15], Zimmermann and Lowenstein [20,38] (resulting in the /incomplete/ acronym BPHZ) is still substantially using the traditional momentum space picture. Even Epstein and Glaser [10], who set the stage for the position space renormalization program based on locality, were proving Lorentz invariance of time-ordered products working in momentum space. It was only in [25] – another famous unpublished preprint of Raymond's – that the problem was translated into a cohomological position space argument (see the historical survey in [13]). This led gradually to viewing renormalization as a problem of extending distributions defined originally for non-coinciding arguments, an approach that, in the words of Stora [30], "from a philosophical point of view, does not require the use – and the removal – of regularizations". The tortuous path from p- to x-space renormalization can be viewed, in modern parlance, as a duality transformation (the good old Fourier integral) mapping a large momentum onto a small distance problem. As relativistic causality does not require the existence of a Poincaré invariant vacuum state, the Stueckelberg–Bogolubov–Epstein–Glaser–Stora position space approach turned out to be the only one suited for the study of perturbative QFT on a curved background (which began flourishing during the last twenty years or so – see [12,16] for recent reviews and references).

Our collaboration started with Raymond reading Sect. 3.2 of the first volume of Hörmander's treatise [17] and pointing out that it is tailor-made for renormalization of a massless theory. It is based on the observation that a density like

$$\mathbf{G}_\ell(x) := G_\ell(x)\frac{d^4x}{\pi^2} = \frac{1}{x^{2\ell}}\frac{d^4x}{\pi^2} \tag{1.1}$$

is a meromorphic distribution valued function of ℓ with simple poles (at $2\ell = 4, 5, 6, \dots$ above). Subtracting the pole term, say at $\ell = 2$, we find a renormalized amplitude G_2^R defined up to a distribution with support at the origin. The ambiguity can be restricted by demanding that this distribution has the same degree of homogeneity as the function G_2 away from the origin (in our case -4). The resulting G_2^R is associate homogeneous of degree -4 and order one. More generally, a logarithmically divergent density $\mathbf{G}$ of an N-dimensional argument $\vec{x}$ defines an *associate homogeneous distribution G of degree $-N$ and order n* if

$$\lambda^N G(\lambda\vec{x}) = G(\vec{x}) + \sum_{j=1}^{n} R_j(G)(\vec{x})\frac{(\ln\lambda)^j}{j!}, \ \lambda > 0, \tag{1.2}$$

where the distributions $R_j(G)$ can be viewed as generalized residues:

$$R_j(G) = Res[(\mathcal{E} + N)^{j-1}G(\vec{x})]\,, \quad \mathcal{E} = \sum_{\alpha=1}^{N} x^\alpha\partial_\alpha, \tag{1.3}$$

satisfying

$$\lambda^N R_j(G)(\lambda\vec{x}) = R_j(G)(\vec{x}) + \sum_{i=j+1}^{n} R_i(G)(\vec{x})\frac{(\ln\lambda)^{i-j}}{(i-j)!}, \ \lambda > 0. \tag{1.4}$$

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