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Chains of topological oscillators with instantons and calculable topological observables in topological quantum mechanics

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Abstract

We extend to a possibly infinite chain the conformally invariant mechanical system that was introduced earlier as a toy model for understanding the topological Yang–Mills theory. It gives a topological quantum model that has interesting and computable zero modes and topological invariants. It confirms the recent conjecture by several authors that supersymmetric quantum mechanics may provide useful tools for understanding robotic mechanical systems (Vitelli et al.) and condensed matter properties (Kane et al.), where trajectories are allowed or not by the conservation of topological indices. The absences of ground state and mass gaps are special features of such systems.

1. Introduction

Topological Quantum Field Theories TQFT's are possible realisations of the invariance under general local field transformations general coordinates invariant symmetries. Such an invariance goes beyond that of current gauge theories. The first non-trivial example of a TQFT was introduced by Witten [1] showing that the genuine $N = 2$ supersymmetric gauge theories contains observables that describe the Donaldson invariants. The reinterpretation [2] of this theory ap-

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peared soon after, as a suitably gauge-fixed quantum field theory stemming from a classical topological invariant that explores the BRST cohomology of general Yang–Mills field transformations modulo ordinary gauge transformations. Because the pattern of TQFT's is that of an ultimate type of gauge-fixing, and because they can be solved, they greatly interested Raymond Stora, who wrote himself a very interesting article on the subject [3].

At the heart of TQFT's, the topological BRST nilpotent operator Q plays a fundamental role. It is such that the TQFT Hamiltonian is basically $H = \frac{1}{2}[Q, \overline{Q}]$. One often defines the physical Hilbert space as the cohomology of Q (states which are annihilated by Q without being the Q transformation of other states). This unambiguous definition of observables from the cohomology of a BRST operator is perfectly suited for the gauge theories of elementary particles (where the expression of Q is more restricted in comparison to that of a TQFT and the relation between Q and H is different). The cohomology of a TQFT is often contained in another cohomology, in which case it is called an equivalent cohomology [2–4]. There were doubts for a while on the validity of this construction of TQFT's, so [5] defined and explored a solvable quantum mechanical supersymmetric example to check very precisely all the details and confirm the construction. The model was that of particle moving in a punctured plane, where the closed trajectories carry topology because of their non-trivial winding numbers. Instantons exist in this case because one choses in this case a potential that yields a supersymmetric action with twisted scale and vector supersymmetries, in fact a superconformal supersymmetry. Strikingly, the ingredients for constructing the model completely reproduce those of the much more involved Yang–Mills topological theory and we completely solved it in [5]. The goal of this article is to generalise this model to a more physical multiparticle case with conformal interactions. To do so, we need first to review [5].

Afterward we will show that [5] can be extended into a very intriguing model, which is quite beautiful and might furthermore have richer applications in practical domains. It is an explicit example of what was foreseen long time ago in [12], for building some robots with rotational constrained degrees of freedom, and more recently by condensed matter physicists, for instance [13]. Our model generalises [5] and gives a sort of conformally vibrating lattice where each site is a particle interacting by superconformal interactions with its nearest neighbours (two in this present case). This model exhibits non-trivial instanton solutions and has some topological observables.

2. The one-particle conformal supersymmetric topological model

The model is a quantum mechanical system of a particle moving in a 2D-plane where one excludes the origin and submitted to a potential we will shortly display. One has a non-trivial topological structure because of trajectories with different possible winding numbers $0 \le N \le \infty$ around the origin. The classical topological symmetry is the group of arbitrary local deformations of each particle trajectory. They can be possibly defined modulo local dilatations of the distance of the particle to the origin. We will see that the model is a conformal one.

We call the time by the real variable t and the Euclidian time by τ, with $t = i\tau$. The cartesian coordinates on the plane are q_i, with $i = 1, 2$, and we often use complex coordinates $z = q_1 + q_2$.

We select trajectories with periodic conditions. Namely, the particle does a closed (multi-)loop between the initial and final times $t = 0$ and $t = T$ (we will choose $T = 1$). An integer winding number $0 \le N \le \infty$ is assigned to all trajectories which can be classified in equivalence classes according to N.

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