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Darkness without dark matter and energy – generalized unimodular gravity

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ABSTRACT

We suggest a Lorentz non-invariant generalization of the unimodular gravity theory, which is classically equivalent to general relativity with a locally inert (devoid of local degrees of freedom) perfect fluid having an equation of state with a constant parameter w. For the range of w near -1 this dark fluid can play the role of dark energy, while for $w = 0$ this dark dust admits spatial inhomogeneities and can be interpreted as dark matter. We discuss possible implications of this model in the cosmological initial conditions problem. In particular, this is the extension of known microcanonical density matrix predictions for the initial quantum state of the closed cosmology to the case of spatially open Universe, based on the imitation of the spatial curvature by the dark fluid density. We also briefly discuss quantization of this model necessarily involving the method of gauge systems with reducible constraints and the effect of this method on the treatment of recently! suggested mechanism of vacuum energy sequestering.

1. Introduction

Dark matter and dark energy phenomena form a dark side of modern precision cosmology and, therefore, represent an unprecedentedly rich playground for various modifications of general relativity (GR). Perhaps, conceptually the most interesting versions of these modifications are the ones which do not involve special types of gravitating matter and originate from the purely metric sector of the theory, like local $f(R)$-gravity or nonlocal cosmology models [1,2]. Usually such modifications are equivalent to adding or removing some local degrees of freedom. Even more interesting is the case when a nontrivial modification occurs without changing the balance of local physical variables – darkness arises without dark energy or dark matter constituents. Known examples of such a concept include, in particular, the unimodular (UM) gravity [3–5], the theory of vacuum energy sequestering [6,7], QCD holonomy mechanism of dark energy [8] and others. Unimodular gravity differs from the Einstein GR by the requirement that at the kinematical level the full set of metric coefficients is subject to the restriction of the unit determinant of the metric tensor. Rather anti-intuitive conclusion that this theory has the same number of local degrees of freedom as GR [9] can be explained by the fact that reduction in the number of independent field variables is compensated by the reduction of the local gauge invariance group, and the main effect of the unimodular modification is the origin of one global degree of freedom playing the role of the cosmological constant.

Extension of the physical sector of the theory by a partial violation of gauge invariance is a well-known and rather popular phenomenon. In particular, reduction from Lorentz symmetry to anisotropic scaling invariance in Lifshitz models is very productive in condensed matter theory context [10], while a similar modification in Horava gravity models [11] opens prospects for renormalizable unitarity preserving gravity theories. Other examples can be found in [12,13]. Here we will consider the synthesis of Lorentz violation with the concept of unimodular gravity [3–5]. This generalized UM gravity incorporates Lorentz violation in the definition of the reduced configuration space of metric coefficients – instead of the requirement of a unit metric determinant this theory is based on the metric field satisfying the following constraint

$$N = N(\gamma), \quad \gamma \equiv \det \gamma_{ij}, \tag{1}$$

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where $N = (-g^{00})^{-1/2}$ is the lapse function and $N(\gamma)$ is some function of γ – the determinant of the spatial metric γ_{ij} in the ADM $(3+1)$-decomposition of metric coefficients $g_{\mu\nu}$,

$$g_{\mu\nu}dx^\mu dx^\nu = (N_i N^i - N^2)\,dt^2 + 2N_i\,dt\,dx^i + \gamma_{ij}dx^i\,dx^j. \tag{2}$$

Here $x^\mu = (t, x^i)$, $\mu = 0,1,2,3$, $i = 1,2,3$ and $N_i = g_{0i}$ is the corresponding shift function.

The motivation for such a generalization of the unimodular gravity is as follows. To begin with, the class of metrics subject to (1) includes the original unimodular theory corresponding to $N(\gamma) = 1/\sqrt{\gamma}$. The right hand side of (1) is invariant under spatial rotations, so that this is a minimal breakdown of Lorentz symmetry from $O(1,3)$ to $O(3)$. Another reason to consider it is an interesting fact that at the classical level such a theory effectively incorporates a special type of matter source – dark fluid with a nonlinear (general barotropic) equation of state. Thus it goes beyond a conventional unimodular gravity by generating the perfect fluid characterized not by just vacuum energy with $p = -\varepsilon$, but by a nontrivial pressure as well. Finally, for a simple class of power-like functions $N(\gamma)$ in (1) it generates an equation of state $p = w\varepsilon$ with a constant w and, moreover, in the comoving reference frame of this fluid renders the density and pressure constant both in space and time.[1] Thus, similarly to the original unimodular gravity it can incorporate as a spacetime constant of motion the analogue of dark energy which is free from clustering but has a constant polytropic parameter w different from -1. In the particular case of a pressureless dust with $w = 0$, corresponding to $N(\gamma) = \text{const}$, the density of this dust is characterized by a single function of spatial coordinates entirely fixed by the initial conditions, which can be interpreted as a model of inhomogeneous distribution of dark matter similar to the mechanism of mimetic model [14].

Here we analyze this model at the classical level and show that on shell (without extra matter sources) it is equivalent to general relativity with this special type of perfect fluid. Its "darkness" can be intuitively interpreted as the absence of *local* degrees of freedom of this fluid, and its effective manifestation can in principle be either the dark energy or dark matter. Rigorous counting its degrees of freedom, which is important for the quantization of this model, requires the analysis of its local gauge invariance. Usual diffeomorphism invariance is obviously broken by the restriction (1) on metric coefficients, which leads to a preferred spacetime foliation by spacelike hypersurfaces. However, there exist reduced diffeomorphisms which leave the theory locally gauge invariant and turn out to be a generalization of volume preserving diffeomorphisms of the unimodular gravity. We briefly discuss them and show that their origin naturally leads to the theory with reducible (linearly dependent) generators. At the quantum level it is subject to Batalin–Vilkovisky technique [16] which allows one to quantize the theory without explicitly disentangling its physical sector.

We conclude the paper by the discussion of how this model can be used within the initial conditions problem in cosmology. Dark fluid of generalized UM gravity can be used to imitate the effect of spatial curvature. This might extend the predictions of the cosmological density matrix construction [17], which are valid only in the spatially closed model, to the phenomenologically more preferable open model with flat space foliation. Another potential application could be the mechanism of sequestering the back reaction effect of quantum vacuum energy recently suggested as a possible solution of Planckian hierarchy and cosmological constant problems [6,7]. Remarkably, the method of careful treatment of the global physical mode responsible for the locally inert dark fluid is the same as that of the sequestering mechanism – the canonical version of the BV method [16], which might clarify acausality puzzles of this mechanism and extend it to noncompact spacetimes.

[1] Since this model violates general coordinate invariance this property of density and pressure becomes frame dependent.

2. Dark fluid and its generalized unimodular invariance

The simplest way to handle the constraint (1) on metric coefficients is not to explicitly substitute it in the Einstein action, but rather incorporate it into the action with the Lagrange multiplier λ,

$$S = \int d^4x \left\{ \frac{M_P^2}{2}\, g^{1/2} R(g) - \lambda \left(\frac{1}{\sqrt{-g^{00}}} - N(\gamma) \right) \right\}. \tag{3}$$

Varying this action with respect to λ and $g_{\mu\nu}$ one obtains the restriction (1) on the metric and the Einstein equation with the perfect fluid matter stress tensor

$$R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R = \frac{1}{M_P^2}\, T^{\mu\nu}, \tag{4}$$

$$\begin{aligned} T^{\mu\nu} &\equiv -\frac{2}{g^{1/2}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x\, \lambda \left(\frac{1}{\sqrt{-g^{00}}} - N(\gamma) \right) \\ &= \varepsilon\, u^\mu u^\nu + p\big(g^{\mu\nu} + u^\mu u^\nu\big), \end{aligned} \tag{5}$$

where the four-velocity $u^\mu = -g^{\mu 0} N$ is a future pointing vector normal to spacelike hypersurfaces of the ADM foliation (2), and its energy density and pressure read

$$\varepsilon = \frac{\lambda}{2\sqrt{\gamma}}, \quad p = \frac{\lambda}{\sqrt{\gamma}} \left(\frac{\gamma}{N} \frac{dN}{d\gamma} \right). \tag{6}$$

Thus, this dark fluid satisfies the equation of state $p = w\varepsilon$ with a generally nonconstant parameter $w = w(\gamma)$ given by

$$w = 2\, \frac{\gamma}{N} \frac{dN}{d\gamma} = 2\, \frac{d\ln N}{d\ln \gamma}. \tag{7}$$

Similarly to the UM gravity [3] the generalized unimodularity condition (1) is not invariant under generic diffeomorphisms of the metric – Lie derivatives with respect to the 4-dimensional vector field ξ^μ which in the $(3+1)$-decomposition can be written down as a column,

$$\delta_\xi g^{\mu\nu} = -\nabla^\mu \xi^\nu - \nabla^\nu \xi^\mu, \quad \xi^\mu = \begin{bmatrix} \xi^0 \\ \xi^i \end{bmatrix}. \tag{8}$$

However, this condition remains invariant under reduced diffeomorphisms with respect to the subset of vector fields $\boldsymbol{\xi}^\mu$ satisfying the equation

$$\begin{aligned} \delta_{\boldsymbol{\xi}} \big(N - N(\gamma)\big)\Big|_{N=N(\gamma)} &= N\big[\,\partial_t \boldsymbol{\xi}^0 - (1+w)\, N^i \partial_i \boldsymbol{\xi}^0 - w\, \partial_i \boldsymbol{\xi}^i\,\big] \\ &= 0, \end{aligned} \tag{9}$$

which in the UM gravity case, $w = -1$, obviously reduces to the equation on parameters of volume preserving diffeomorphisms $\partial_\mu \boldsymbol{\xi}^\mu = 0$ [3].

With the decomposition of ξ^i into the longitudinal and transverse parts,[2]

$$\boldsymbol{\xi}^i = \sqrt{\gamma}\,\big(\gamma^{ij}\partial_j \varphi + \xi_\perp^i\big), \quad \partial_i(\sqrt{\gamma}\,\xi_\perp^i) = 0, \tag{10}$$

[2] Since general diffeomorphism invariance is broken, the transformation properties of φ and $\xi_\perp^i$ are no longer of a scalar and vector type, and the $\sqrt{\gamma}$-factor is added merely for reasons of convenience.