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Absorption of electromagnetic and gravitational waves by Kerr black holes

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A R T I C L E I N F O

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A B S T R A C T

We calculate the absorption cross section for planar waves incident upon Kerr black holes, and present a unified picture for scalar, electromagnetic and gravitational waves. We highlight the spin-helicity effect that arises from a coupling between the rotation of the black hole and the helicity of a circularly-polarized wave. For the case of on-axis incidence, we introduce an extended 'sinc approximation' to quantify the spin-helicity effect in the strong-field regime.

1. Introduction

Black holes, once dismissed as a mathematical artifact of Einstein's theory of general relativity (GR), have come to play a central role in modern astronomy and theoretical physics [1,2]. In astronomy, black holes provide a solution: in galaxy formation scenarios, in active galactic nuclei and in core-collapse supernovae, for instance. In theoretical physics, black holes pose a challenge: as spacetime curvature grows without bound in GR, the classical theory breaks down. Yet, novel quantum gravity effects apparently remain shrouded by a horizon endowed with generic thermodynamic properties [3].

Two recent advances in interferometry have opened new data channels on astrophysical black holes. In September 2015, LIGO detected the first gravitational-wave signal: a characteristic 'chirp' from a black hole binary merger [4]. Hundreds more chirps are anticipated over the next decade [5]. In April 2017, the Event Horizon Telescope (EHT) [6] – a global array of radio telescopes linked by very long baseline interferometry – observed the supermassive black hole candidates Sgr. A* and M87* at a resolution three orders of magnitude beyond that of the Hubble telescope [7]. Ultimately, the EHT will seek to study the black hole shadow itself [8–10], using techniques to surpass the diffraction limit [11].

These experimental advances motivate study of the interaction of electromagnetic waves (EWs) and gravitational waves (GWs) with black holes [12–14]. EWs and GWs propagating on curved spacetimes in vacuum share some traits. For example, both possess two independent (transverse) polarizations that are parallel-transported along null geodesics in the ray-optics limit. Yet there are key physical differences. GWs are tenuous, in the sense that they are not significantly attenuated or rescattered by matter sources. GWs are typically long-wavelength and polarized, because rotating quadrupoles (for example, binary systems or asymmetric neutron stars) predominantly emit circular-polarized waves at twice the rotational frequency [15]. For example, $\lambda \sim 10^{-3}$ m for EHT observations, whereas $\lambda \sim 10^{7}$ m for GW150914.

In this Letter we examine the absorption of a monochromatic planar wave of frequency ω incident upon a Kerr black hole of mass M and angular momentum J in vacuum. We calculate the absorption cross section $\sigma_{\rm abs}$, i.e., the cross-sectional area of the black hole shadow [8–10] beyond the ray-optics approximation. For the first time, we present unifying results for scalar ($s = 0$), electromagnetic ($s = 1$) and gravitational ($s = 2$) waves. Our results highlight the influence of two key phenomena: superradiance and the spin-helicity effect, described below.

The absorption scenario, illustrated in Fig. 1, is encapsulated by several dimensionless parameters: the ratio of the gravitational length to the (reduced) wavelength $GM\omega/c^3$; the dimensionless black hole spin $a^* \equiv a/M$ where $a = Jc^2/GM$ $(0 \le a^* < 1)$; the spin of the field $s = 0, 1, 2$; the angle of incidence with respect to the black hole axis γ; and the helicity of the circular polarization ± 1. We adopt natural units such that $G = c = 1$.

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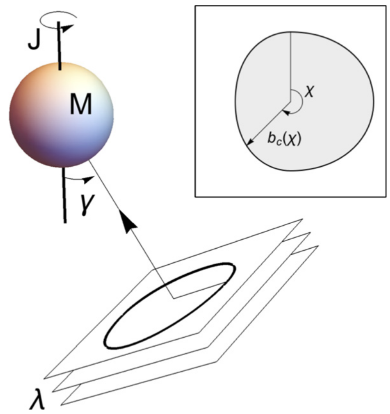

Fig. 1. A planar wave of frequency $\omega = 2\pi c/\lambda$ incident upon a rotating black hole of mass M and angular momentum J at an angle γ. *Inset:* the locus $b_c(\chi)$ of the black hole shadow on the wavefront.

2. Concepts

2.1. Black hole shadows

An observer studying a black hole in vacuum with a pinhole camera will see a dark region on the image plane defined by the set of null-geodesic rays entering the pinhole which, when traced backwards in time, pass into the black hole. The boundary of the shadow is determined by those rays which asymptote towards an (unstable) photon orbit, defining an angular radius $\alpha(\chi)$ in terms of the projection angle χ. Alternatively, a shadow can be defined on a planar surface in terms of an impact parameter $b(\chi)$, using those rays orthogonal the surface, as shown in Fig. 1. Far from the black hole, there is an approximately linear relationship $b(\chi) = r_0\alpha(\chi) + O\left(\frac{GM}{c^2 r_0}\right)$; the two approaches are closely related. Here we extend the latter approach to consider monochromatic waves of a finite wavelength.

In the geometrical-optics limit ($\lambda \to 0$), an observer at radial coordinate r_0 sees a shadow of angular radius α where [16]

$$\sin^2\alpha = \frac{27}{4}\frac{(\rho-1)}{\rho^3}, \qquad \rho \equiv \frac{r_0 c^2}{GM}. \tag{1}$$

For Sgr A*, $\alpha \approx 25$ µarcsec, with $r_0 \approx 8.3$ kpc and $M \approx 4.1 \times 10^6\ M_\odot$ [17]. In Kerr spacetime, α is a function of angle χ relative to the (projected) spin axis.

Here we seek to study Kerr shadows beyond the geometrical-optics regime. We shall focus on the difference between $\sigma_{\text{abs}}(\omega)$, the absorption cross section at fixed frequency ω, and the σ_{geo}, the geometric cross section defined by

$$\sigma_{\text{geo}} = \tfrac{1}{2}\int_0^{2\pi} b_c^2(\chi)d\chi. \tag{2}$$

2.2. Superradiance and spin-helicity

Superradiance is a radiation-enhancement mechanism by which a black hole may shed mass and angular momentum and yet still increase its horizon area, and thus its entropy [18]. As a consequence, σ_{abs} may become negative at low frequencies, through stimulated emission. The effect is strongly enhanced by spin s.

The spin-helicity effect is a coupling between a rotating source, such as a Kerr black hole, and the helicity of a polarized wave of finite wavelength λ [19]. A rotating spacetime distinguishes and separates waves of opposite helicity [20–22]. In the weak-field, rays are deflected through an angle $\zeta\Theta_E$, with $\Theta_E \equiv \frac{4GM}{c^2 b}$ the Einstein angle and $\zeta = 1 + \ldots$ an asymptotic series in which the spin-helicity effect is anticipated at $O\left(\frac{J\lambda}{Mcb^2}\right)$ [19]. In the strong-field, we anticipate that waves with a counter-rotating circular polarization are preferentially absorbed ($\sigma_{\text{abs}}^- > \sigma_{\text{abs}}^+$).

3. Method

3.1. Waves on the Kerr spacetime

The Kerr spacetime is described in Boyer–Lindquist coordinates $\{t, r, \theta, \phi\}$ by the line element

$$\begin{aligned} ds^2 = &-\frac{1}{\Sigma}(\Sigma - 2Mr)\,dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}dt\,d\phi + \frac{\Sigma}{\Delta}dr^2 \\ &+ \Sigma d\theta^2 + \frac{(r^2+a^2)^2\sin^2\theta - \Delta a^2\sin^4\theta}{\Sigma}d\phi^2, \end{aligned} \tag{3}$$

where $\Sigma \equiv r^2 + a^2\cos^2\theta$, and $\Delta \equiv r^2 - 2Mr + a^2$. We focus on the $a^2 < M^2$ case of a rotating BH with two distinct horizons: an internal (Cauchy) horizon located at $r_- = M - \sqrt{M^2 - a^2}$ and an external (event) horizon at $r_+ = M + \sqrt{M^2 - a^2}$.

In the vicinity of a Kerr black hole, perturbing fields are described by a single master equation, first obtained by Teukolsky [23] using the Newman–Penrose formalism. In vacuum the master equation takes the form

$$\begin{aligned} &\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\phi} \\ &+ \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\phi^2} - \Delta^{-\mathfrak{s}}\frac{\partial}{\partial r}\left(\Delta^{\mathfrak{s}+1}\frac{\partial\phi}{\partial r}\right) \\ &- \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + (\mathfrak{s}^2\cot^2\theta - \mathfrak{s})\psi \\ &- 2\mathfrak{s}\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\phi} \\ &- 2\mathfrak{s}\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} = 0, \end{aligned} \tag{4}$$

where $\mathfrak{s}$ is the spin-weight of the field. We use $\mathfrak{s} = -s$ throughout, where $s = 0, 1, 2$ for scalar, electromagnetic and gravitational fields, respectively. One can separate variables in Eq. (4) using the standard ansatz

$$\psi_{\mathfrak{s}lm\omega}(t, r, \theta, \phi) = R_{\mathfrak{s}lm\omega}(r)S_{\mathfrak{s}lm\omega}(\theta)e^{-i(\omega t - m\phi)}, \tag{5}$$

to obtain angular and radial equations,

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS_{\mathfrak{s}lm\omega}}{d\theta}\right) + U_{\mathfrak{s}lm\omega}(\theta)S_{\mathfrak{s}lm\omega} = 0, \tag{6}$$

$$\Delta^{-\mathfrak{s}}\frac{d}{dr}\left(\Delta^{\mathfrak{s}+1}\frac{dR_{\mathfrak{s}lm\omega}}{dr}\right) + V_{\mathfrak{s}lm\omega}(r)R_{\mathfrak{s}lm\omega} = 0, \tag{7}$$

where

$$\begin{aligned} U_{\mathfrak{s}lm\omega} &\equiv \lambda_{\mathfrak{s}lm\omega} + 2am\omega - 2a\omega\mathfrak{s}\cos\theta - \frac{(m + a\cos\theta)^2}{\sin^2\theta} + \mathfrak{s}, \\ V_{\mathfrak{s}lm\omega} &\equiv \frac{1}{\Delta}\left[K^2 - 2(r-M)K\right] - \lambda_{\mathfrak{s}lm\omega} + 4i\omega\mathfrak{s}r, \end{aligned} \tag{8}$$

and $K \equiv (r^2 + a^2)\omega - am$. The angular functions $S_{\mathfrak{s}lm\omega}(\theta)$ are known as spin-weighted spheroidal harmonics, and have as limiting cases the spheroidal harmonics ($\mathfrak{s} = 0$) and the spin-weighted spherical harmonics ($a\omega = 0$).