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Rapidity distribution of particle multiplicity in DIS at small x

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ABSTRACT

Analytical study of the rapidity distribution of the final state particles in deep inelastic scattering at small x is presented. We separate and analyze three sources of particle production: fragmentation of the quark–antiquark pair that these an impact, accompanying coherent soft gluon radiation due to octet color exchange in the t-channel, and fragmentation of gluons that form parton distribution functions.

Connection to Catani–Ciafaloni–Fiorani–Marchesini (CCFM) equations and the role of gluon reggeization are also discussed.

1. Introduction

Perturbative QCD approach successfully describes internal structure of quark and gluon jets produced in e^+e^- annihilation into hadrons.

The final states of hard lepton–hadron and hadron–hadron collisions have a more complicated structure. In this case, apart from jets formed by the partons originating from the underlying hard scattering, fragmentation of the initial state hadron(s) also contributes to the particle yield.

Really, multiparticle production in hard processes is driven by radiation and successive cascading of relatively soft gluons. Soft bremsstrahlung is subject to coherence effects. Analysis of such effects in jets has led to "angular ordering" (AO) as means of organizing parton multiplication in terms of probabilistic time-like cascades [1]. For the space-like case the corresponding problem was addressed and solved by L. Gribov et al. in [2,3] and independently by M. Ciafaloni [4].

It was shown that in order to formulate the probabilistic picture of the gluon radiation caused by fragmentation of a space-like parton ensemble, one has to impose AO as well, similar to the time-like jet evolution case. This result was further developed by S. Catani, F. Fiorani and G. Marchesini, and laid ground for the CCFM scheme for generating final states of small-x DIS processes in accord with the perturbative QCD [5].

In this letter we study pseudorapidity (angular) distribution of particles produced in small-x DIS processes.

Similar to [2] where the inclusive energy spectra of final state particles were derived, we find three essential contributions to the answer. The first one ($dn^{(1)}$) originates from fragmentation of the struck quark and its partner antiquark (at small x it is a sea $q\bar{q}$ pair that is hit by an incident lepton). This contribution dominates the particle yield at large momentum transfer $q^2 = -Q^2$. Two more contributions, formally subleading but rather important, are due to the underlying space-like gluon cascade that produces the quark pair. One of them ($dn^{(2)}$) is driven by coherent soft gluon radiation caused by octet t-channel color exchange, and the last one ($dn^{(3)}$) — by fragmentation of relatively hard (energetic) gluons that determine the hadron structure functions.

Section 2 is devoted to derivation of corresponding analytic expressions for the spectrum of final particles. In Section 3 we present numerical results for pseudorapidity distribution in DIS for different values of x and Q^2, both academic and realistic.

2. Approximation and main contributions

Collinear approximation allows one to construct a probabilistic QCD cascade picture of multiparticle production and, in particular, to separate initial and final state radiation. Selecting and resuming contributions in which each power of α_s is accompanied by a logarithmic integration over parton transverse momentum, gives rise to the Leading Logarithmic Approximation (LLA).

Because of the double-logarithmic nature of gluon radiation, the true perturbative expansion parameter for observables like *par-*

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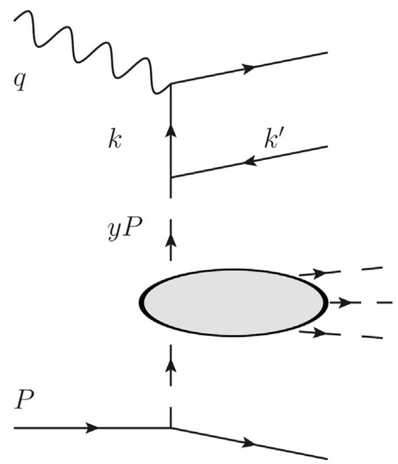

Fig. 1. Gluon ladder attached to the quark box.

ticle multiplicity turns out to be not the QCD coupling α_s itself, but rather $\sqrt{\alpha_s}$. So, the next-to-leading logarithmic approximation (NLLA) effects are down by $\sqrt{\alpha_s}$ as compared with the LLA, etc.

The general NLLA formulae describing the sub-jet structure of the DIS final state in terms of generating functionals were presented in the paper that has introduced the $k_\perp$-clustering algorithm for jets in DIS and hadron–hadron collisions [6].

This approximation, however, turns out to be insufficient to access the structure of the fragmentation of a target proton in DIS. This region provides a $\mathcal{O}(\alpha_s)$ fraction of the total particle yield, and therefore is formally of the NNLL nature. At the same time, this kinematical region widens and becomes important for small values of Bjorken x. In [2] it was demonstrated that the perturbative QCD analysis could be carried out, and NNLA expressions could be derived, if one looks upon $\ln 1/x$ as an additional enhancement, and selects specific NNLL contributions $\mathcal{O}(\alpha_s \ln 1/x)$ in each order of the pQCD expansion. Within this approach approximate analytic expressions for the inclusive *energy* spectrum of final particles has been derived. In this letter we extend the analysis of [2] to the case of the *pseudorapidity* (angular) distribution of particles produced in small-x DIS processes.

We will treat the process in the Breit reference frame ($\mu = 0$, $-q^2 = Q^2$, $x = Q/2P \ll 1$). In this frame proton fragmentation occupies positive pseudorapidities (target fragmentation region)

$$\eta = -\ln\frac{\vartheta}{2},$$

with ϑ the particle production angle with respect to the proton direction. The struck quark jet populates the region $\eta < 0$ (current fragmentation region).

DIS cross section at small x is dominated by space-like parton fluctuations that have the structure of a gluon ladder attached to the quark box as shown in Fig. 1.

2.1. Quark box

Fragmentation of the quark k' in Fig. 1 gives rise to a jet with an opening angle $\Theta' \simeq k'_t/\beta' P$. Here Θ' is the quark production angle, and k'_t and β' — its transverse momentum and longitudinal momentum fraction (for definition of the Sudakov decomposition of parton momenta see Appendix A). This should be taken together with radiation off the virtual quark line k in the interval of Breit-frame angles $\Theta' \le \vartheta$, thus forming a full quark jet in the target fragmentation region (proton hemisphere), similar to that in $e^+e^- \to q\bar{q}$ annihilation with invariant annihilation energy $s = Q^2$.

Similarly, the struck quark with momentum $q+k$ produces the second jet populating the current fragmentation (photon hemisphere).

Measuring pseudorapidity of a final particle introduces certain competition at the level of collinear logarithmic enhancements.

Consider a gluon with positive pseudorapidity. It can belong to the jet that the quark with momentum k' develops in the final state. In this case, logarithmic integration over the relative angle $\Theta_{k',\ell}$ between the three-momenta of the quark, $\vec{k}'$, and that of the gluon, $\vec{\ell}$, runs in the kinematical region

$$\Theta_{k',\ell} \ll \Theta' \simeq \vartheta. \qquad (1)$$

The energy of the quark k' is typically of the order of $xP = Q/2$. Thus, logarithmic integrations over the gluon energy ℓ and the relative angle (1) produce the total quark jet multiplicity factor at a hardness scale $Q\vartheta/2$,

$$k'_t \simeq k'_0 \cdot \Theta' \sim \frac{Q}{2}\vartheta, \quad \mathcal{N}_q\left(Q\sin\frac{\vartheta}{2}\right).$$

At the same time, the fact that the quark transverse momentum is fixed, corresponds to taking logarithmic derivative of the quark pdf $D_h^q(x;\mu^2)$. This gives rise to a contribution

$$\mathcal{N}_q\left(Q\sin\frac{\vartheta}{2}\right)\cdot\frac{d}{d\eta}D_h^q\left(x;Q^2\sin^2\frac{\vartheta}{2}\right). \qquad (2a)$$

Another logarithmic enhancement may originate from the integral over transverse momentum of the quark in the alternative region of production angles, namely

$$\Theta' \ll \vartheta \simeq \Theta_{k',\ell}.$$

This integration gives rise to the quark pdf at the same scale $Q\vartheta/2$, while the multiplicity flow at a fixed angle ϑ is described by the derivative:

$$D_h^q\left(x;Q^2\sin^2\frac{\vartheta}{2}\right)\cdot\frac{d}{d\eta}\mathcal{N}_q\left(Q\sin\frac{\vartheta}{2}\right). \qquad (2b)$$

The two contributions (2) combine into

$$\frac{d}{d\eta}\left[D_h^q\left(x;Q^2\sin^2\frac{\vartheta}{2}\right)\mathcal{N}_q\left(Q\sin\frac{\vartheta}{2}\right)\right] \qquad \eta > 0. \qquad (3)$$

Analogous consideration applies to the radiation in the current fragmentation region, $\eta < 0$ (with replacement of ϑ by $\pi - \vartheta$).

Finally, for entire pseudorapidity region we get an elegant expression

$$D_h^q(x;Q^2)\frac{dn^{(1)}}{d\eta} = \frac{d}{d\eta}\left[D_h^q\left(x;Q^2\sin^2\frac{\vartheta}{2}\right)\mathcal{N}_q\left(Q\sin\frac{\vartheta}{2}\right) + D_h^q\left(x;Q^2\cos^2\frac{\vartheta}{2}\right)\mathcal{N}_q\left(Q\cos\frac{\vartheta}{2}\right)\right]. \qquad (4)$$

Integrating over pseudorapidity, for accompanying particle multiplicity due to radiation off the quark box we obtain

$$n^{(1)} = 2\mathcal{N}_q(Q),$$

which expression coincides with the mean multiplicity in $e^+e^- \to q\bar{q}$ with invariant annihilation energy $s = Q^2$.

2.2. Soft *t*-channel radiation

If the quark box particle production is similar to that in $e^+e^- \to q\bar{q}$, in small-x DIS there is an additional essential source of final particles that mimics a *gluon* jet. It is due to coherent radiation of soft gluons ℓ with longitudinal momenta $\beta_\ell < x$, and emission angles *smaller* that the production angle of the quark k':