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Gravitational instabilities of the cosmic neutrino background with non-zero lepton number

Neil D. Barrie, Archil Kobakhidze

ARC Centre of Excellence for Particle Physics at the Terascale, School of Physics, The University of Sydney, NSW 2006, Australia

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A B S T R A C T

We argue that a cosmic neutrino background that carries non-zero lepton charge develops gravitational instabilities. Fundamentally, these instabilities are related to the mixed gravity-lepton number anomaly. We have explicitly computed the gravitational Chern–Simons term which is generated quantum-mechanically in the effective action in the presence of a lepton number asymmetric neutrino background. The induced Chern–Simons term has a twofold effect: (i) gravitational waves propagating in such a neutrino background exhibit birefringent behaviour leading to an enhancement/suppression of the gravitational wave amplitudes depending on the polarisation, where the magnitude of this effect is related to the size of the lepton asymmetry; (ii) Negative energy graviton modes are induced in the high frequency regime, which leads to very fast vacuum decay producing, e.g., positive energy photons and negative energy gravitons. From the constraint on the present radiation energy density, we obtain an interesting bound on the lepton asymmetry of the universe.

1. Introduction

Along with the Cosmic Microwave Background radiation (CMB), the existence of the Cosmic Neutrino Background (CνB) is an inescapable prediction of the standard hot big bang cosmology (see e.g. [1] for a review). It is assumed to be a highly homogeneous and isotropic distribution of relic neutrinos with the temperature:

$$T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \approx 1.945 \text{ K} , \quad (1)$$

where $T_\gamma = 2.725$ K is the temperature of the CMB today. Unlike the CMB though, the CνB is extremely hard to detect and its properties are largely unknown. Namely, the CνB may exhibit a neutrino–antineutrino asymmetry

$$\eta_{\nu_\alpha} = \frac{n_{\nu_\alpha} - \bar{n}_{\nu_\alpha}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)}\left(\xi_\alpha + \frac{\xi_\alpha^3}{\pi^2}\right) , \quad (2)$$

for each neutrino flavour $\alpha = e, \ \mu, \ \tau$. Here $\xi_\alpha = \mu_\alpha/T$ is the degeneracy parameter, μ_α being the chemical potential for α-neutrinos. In fact, such an asymmetry is generically expected to be of the order of the observed baryon–antibaryon asymmetry, $\eta_B = (n_B - \bar{n}_B)/n_\gamma \sim 10^{-10}$, due to the equilibration by sphalerons of lepton and baryon asymmetries in the very early universe. However, there are also models [2,3] which predict an asymmetry in the neutrino sector that are many orders of magnitude larger than η_B. If so, this would have interesting cosmological implications for the QCD phase transition [4] and/or large-scale magnetic fields [5].

The most stringent bound on the neutrino asymmetry comes from the successful theory of big bang nucleosynthesis (BBN). BBN primarily constrains the electron neutrino asymmetry. However, this bound applies to all flavours, since neutrino oscillations below ∼10 MeV are sizeable enough to lead to an approximate flavour equilibrium before BBN, $\mu_e \approx \mu_\mu \approx \mu_\tau (\equiv \mu_\nu)$ [6–8].[1] The updated analysis presented in [9] leads to the following bound on the common degeneracy parameter:

$$|\xi_\nu| \lesssim 0.049 \quad (3)$$

In this paper, we argue that the lepton asymmetry in the active neutrino sector leads to gravitational instabilities. These instabilities originate from the gravity-lepton number chiral quantum

E-mail addresses: neil.barrie@sydney.edu.au (N.D. Barrie), archilk@physics.usyd.edu.au (A. Kobakhidze).

[1] See, however, a recent analysis in [10], where a larger η_{ν_μ,ν_τ} asymmetry is found to be allowed.

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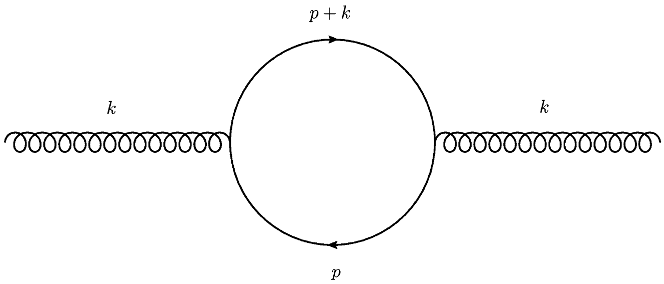

Fig. 1. Parity violating contribution to the fermion propagator.

anomaly, which is present in the Standard Model when considering Majorana neutrinos. Indeed, in the case of Majorana neutrinos, a non-zero lepton asymmetry for active neutrinos implies an imbalance between neutrinos of left-handed chirality and antineutrinos of right-handed chirality which, as we demonstrate explicitly below, leads to the inducement of the gravitational Chern–Simons term in the effective action. This is analogous to the inducement of Chern–Simons terms in gauge theories [11].

The induced Chern–Simons term causes birefringence of gravitational waves propagating in the lepton asymmetric neutrino background, which can be sizeable for gravitational waves generated at very early times. More importantly, short-scale gravitational fluctuations exhibit negative energy modes, which lead to a rapid decay of the vacuum state, e.g., into negative energy graviton and photons. Since the graviton energy is not bounded from below, the phase space for this process is formally infinite, that is, the instability is expected to develop very rapidly. Conservatively, we introduce a comoving cut-off Λ and compute the spectrum of produced photons as a function of neutrino chemical potential. From the constraint on the radiation energy density today, we then obtain an interesting bound on the neutrino degeneracy parameter:

$$\xi_\nu \lesssim 2\cdot 10^{-41}\left(\frac{T_a}{10^{15}\,\mathrm{GeV}}\right)^{4/3}\left(\frac{M_P}{\Lambda}\right)^{17/3}, \tag{4}$$

provided that the lepton asymmetry has been generated above $T_* \gtrsim \frac{440}{\sqrt{\xi}}\sqrt{M_P/\Lambda}$ GeV (here $M_P \approx 2.4\cdot 10^{18}$ GeV is the reduced Planck mass), where T_a is the temperature at which the asymmetry is generated.

This paper is organised as follows; in Section 2 we describe the calculation of the graviton polarisation tensor in the presence of a lepton asymmetric CνB, and consider the associated effective action. Section 3 illustrates the birefringent behaviour of gravitational waves in such a background, while in Section 4 we derive constraints on the CνB lepton asymmetry through the induced gravitational instabilities, before concluding in Section 5.

2. Graviton polarisation tensor in the lepton asymmetric CνB

We calculate the inducement of the Chern–Simons like terms in the effective graviton Lagrangian through the 1-loop graviton polarization diagram depicted in Fig. 1, influenced by a lepton asymmetric neutrino background. The lepton asymmetry is enforced in the Lagrangian through a chiral chemical potential $\mathcal{L}_{\mu_\nu} = \bar{\nu}\not{b}\gamma^5\nu = \mu_\nu\bar{\nu}\gamma_0\gamma^5\nu$, for which we have considered the frame in which the CνB is at rest ($\not{b} = \mu_\nu\gamma_0$). The neutrino propagator is altered as follows:

$$S(p) = \frac{i}{\not{p} - m - \not{b}\gamma^5} = \frac{i}{\not{p} - m}\sum_{n=0}^{\infty}\left(-i\not{b}\gamma^5\frac{i}{\not{p}-m}\right)^n$$
$$\equiv S_0(p) + \sum_{n=1}^{\infty} S_n(p)\,, \tag{5}$$

where $S_0(p)$ is the usual fermion propagator in vacuum. The above modified neutrino propagator to first-order in μ_ν is given by $S(p) \approx S_0(p) - i\mu_\nu\frac{i}{\not{p}-m}\gamma_0\gamma^5\frac{i}{\not{p}-m}$. The higher order terms in b_μ, or μ, are neglected because we are only interested in the linear terms in b_μ, which will result in a Chern–Simons like term. Taking this, and using the standard Feynman rules, we find that the parity odd part of the full graviton polarization tensor is:

$$\Pi_{\mu\nu\rho\sigma} = -\int\frac{d^4p}{(2\pi)^4}(2p+k)_\nu(2p+k)_\sigma$$
$$\times\Big[Tr(\gamma_\mu S_0(p+k)\gamma_\rho S_1(p))$$
$$+ Tr(\gamma_\rho S_0(p)\gamma_\mu S_1(p+k))\Big]$$
$$+(\mu\leftrightarrow\nu)+(\rho\leftrightarrow\sigma)+(\mu\leftrightarrow\nu,\rho\leftrightarrow\sigma)\,. \tag{6}$$

To evaluate the divergent loop integral in (6) we employ the dimensional regularization method ($d = 4-\epsilon$, $\epsilon\to 0$) and utilise the relations given in Appendix A. We hence obtain:

$$\Pi_{\mu\nu\rho\sigma} = \frac{\mu_\nu}{8\pi^2}k^\alpha\varepsilon_{\mu\rho\alpha 0}\int_0^1 dx\left[\frac{4\pi^2\lambda^2}{M^2}\right]^\epsilon$$
$$\times\Big[8x^2(1-x)^2(1-2x)^2\frac{k^2}{M^2}\Gamma(1+\epsilon)k_\nu k_\sigma$$
$$+(24x^2-44x+18)\Gamma(\epsilon-1)M^2\eta_{\nu\sigma}$$
$$-16x^2(1-x)^2\Gamma(\epsilon)k^2\eta_{\nu\sigma}$$
$$-(80x^4-192x^3+156x^2-50x+5)\Gamma(\epsilon)k_\nu k_\sigma\Big] \tag{7}$$
$$+(\mu\leftrightarrow\nu)+(\rho\leftrightarrow\sigma)+(\mu\leftrightarrow\nu,\rho\leftrightarrow\sigma)\,, \tag{8}$$

where $M^2 = m^2 - x(1-x)k^2$ and the limit $\epsilon\to 0$ has been assumed. In simplifying this result we find a divergent quantity of the following form:

$$\Pi^{(div)}_{\mu\nu\rho\sigma} = -\frac{1}{\epsilon}\frac{\mu_\nu}{2\pi^2}k^\alpha\varepsilon_{\mu\rho\alpha 0}m^2\eta_{\nu\sigma}$$
$$+(\mu\leftrightarrow\nu)+(\rho\leftrightarrow\sigma)+(\mu\leftrightarrow\nu,\rho\leftrightarrow\sigma)\,, \tag{9}$$

where γ is Euler's constant. A straightforward inspection reveals that this divergent term does not satisfy the gravitational Ward identity, $k^\nu\Pi^{(div)}_{\mu\nu\rho\sigma}\neq 0$, and hence violates the gauge invariance of the effective gravitational action. This has also been observed previously in a somewhat related calculation in Ref. [12]. The origin of this violation is rooted in the method of dimensional regularization, which violates Local Lorentz invariance explicitly through the extrapolation to non-integer spacetime dimensions $d = 4-\epsilon$. Therefore, following the standard lore, we introduce non-invariant counter-terms to renormalise away this unphysical divergent term. The polarisation tensor then takes the following simple form:

$$\Pi_{\mu\nu\rho\sigma} = \mu_\nu\varepsilon_{\mu\rho\alpha 0}k^\alpha[k_\nu k_\sigma - k^2\eta_{\nu\sigma}]C(k^2)$$
$$+(\mu\leftrightarrow\nu)+(\rho\leftrightarrow\sigma)+(\mu\leftrightarrow\nu,\rho\leftrightarrow\sigma)\,, \tag{10}$$

where

$$C(k^2) = \frac{1}{192\pi^2} - \frac{m^2}{16\pi^2(k^2)^{3/2}}$$
$$\times\left[\sqrt{k^2} - \sqrt{4m^2-k^2}\tan^{-1}\left(\frac{\sqrt{k^2}}{\sqrt{4m^2-k^2}}\right)\right]. \tag{11}$$

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