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Quantum quench and scaling of entanglement entropy

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ABSTRACT

Global quantum quench with a finite quench rate which crosses critical points is known to lead to universal scaling of correlation functions as functions of the quench rate. In this work, we explore scaling properties of the entanglement entropy of a subsystem in a harmonic chain during a mass quench which asymptotes to finite constant values at early and late times and for which the dynamics is exactly solvable. When the initial state is the ground state, we find that for large enough subsystem sizes the entanglement entropy becomes independent of size. This is consistent with Kibble–Zurek scaling for slow quenches, and with recently discussed "fast quench scaling" for quenches fast compared to physical scales, but slow compared to UV cutoff scales.

1. Introduction

The behavior of entanglement of a many-body system that undergoes a quantum quench has been a subject of great interest in recent times. When the quench is instantaneous (i.e. a sudden change of the hamiltonian), several results are known. Perhaps the best known result pertains to the entanglement entropy (EE) of a region of size l in a $1+1$ dimensional conformal field theory following a global instantaneous quench, $S_{EE}(l)$. As shown in [1], $S_{EE}(l)$ grows linearly in time till $t \approx l/2$ and then saturates to a constant value typical of a thermal state – a feature which has been studied extensively in both field theory and in holography. Generalizations of this result to conserved charges and higher dimensions have been discussed more recently [2–4]. The emphasis of these studies is to probe the *time evolution* of the entanglement entropy.

In physical situations, quantum quench has a finite rate, characterized by a time scale δt, that can vary from very small to very large. When the quench involves a critical point, universal scaling behavior has been found for correlation functions at *early* times. The most famous scaling appears for a global quench which starts from a massive phase with an initial gap m_g, crosses a critical point (chosen to be e.g. at time $t=0$) and ends in another massive phase. For *slow quenches* (large δt), it has been conjectured that quantities obey Kibble–Zurek scaling [5]: evidence for this has been found in several solvable models and in numerical simulations [6,7]. Such scaling follows from two assumptions. First, it is assumed that as soon as the initial adiabatic evolution breaks down at some time $-t_{KZ}$ (the Kibble–Zurek time) the system becomes roughly diabatic. Secondly, one assumes that the only length scale in the critical region is the instantaneous correlation length ξ_{KZ} at the time $t=-t_{KZ}$. This implies that, for example, one point functions scale as $\langle\mathcal{O}(t)\rangle \sim \xi_{KZ}^{-\Delta}$, where Δ denotes the conformal dimension of the operator $\mathcal{O}$ at the critical point. An improved conjecture involves scaling functions. For example, one and two point correlation functions are expected to be of the form [8–14]

$$\langle\mathcal{O}(t)\rangle \sim \xi_{KZ}^{-\Delta}\, F(t/t_{KZ})$$
$$\langle\mathcal{O}(\vec{x},t)\mathcal{O}(\vec{x}',t')\rangle \sim \xi_{KZ}^{-2\Delta} F\left[\frac{|\vec{x}-\vec{x}'|}{\xi_{KZ}}, \frac{(t-t')}{t_{KZ}}\right] \tag{1}$$

Some time ago, studies of slow quenches in AdS/CFT models have led to some insight into the origin of such scaling without making these assumptions [15].

For protocols in relativistic theories which asymptote to constant values at early times, one finds a different scaling behavior in the regime $\Lambda_{UV}^{-1} \ll \delta t \ll m_{phys}^{-1}$, where Λ_{UV} is the UV cutoff scale, and m_{phys} denotes any physical mass scale in the problem. For example,

$$\langle\mathcal{O}(t)\rangle \sim \delta t^{d-2\Delta} \tag{2}$$

where d is the space–time dimension. This "fast quench scaling" behavior was first found in holographic studies [16] and subse-

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quently shown to be a completely general result in any relativistic quantum field theory [17]. The result follows from causality, and the fact that in this regime linear response becomes a good approximation. Finally, in the limit of an instantaneous quench, suitable quantities saturate as a function of the rate: for quench to a critical theory a rich variety of universal results are known in $1+1$ dimensions [18].

Much less is known about the behavior of entanglement and Renyi entropies *as functions of the quench rate* – a key ingredient of universality. This has been studied for the 1d Ising model (and generalizations) with a transverse field which depends *linearly* on time, $g(t) = 1 - \frac{t}{\tau_Q}$ [19,9,20]. The system is prepared in the instantaneous ground state at some initial time, crossing criticality at $t=0$. The emphasis of [19] and [9,20] is on the slow regime, which means $\tau_Q \gg a$ where a is the lattice spacing, while [21] also studies smaller values of τ_Q. In particular, [19] and [21] studied the EE for half of a finite chain and found that the answer approaches $S_{EE} \sim \frac{1}{12}\log \xi_{KZ}$ after sufficiently slow quenches. This is consistent with the standard assumptions which lead to Kibble–Zurek scaling mentioned above. According to these assumptions, the system evolves adiabatically till $t = -t_{KZ}$ and enters a phase of diabatic evolution soon afterwards. Thus the state of the system at $t=0$ is not far from the ground state of the instantaneous hamiltonian at $t = -t_{KZ}$. Furthermore when $\tau_Q \gg 1$ in lattice units, ξ_{KZ} is large, and the instantaneous state is close to criticality. In such a state, the entanglement entropy of a subregion of a large chain with N_A boundary points should obey an "area law" $\frac{c}{6}N_A \log(\xi_{KZ})$, where c is the central charge. When the subsystem is half space $N_A = 1$ and for the Ising model the central charge is $c = 1/2$. Similarly, [9,20] studied the EE of a subsystem of finite size l in an infinite 1d Ising model, with a transverse field linear in time, starting with the ground state at $t = -\infty$. The EE close to the critical point for $l \gg \xi_{KZ}$ was found to saturate to $S_{EE} = (\text{constant}) + \frac{1}{6}\log(\kappa(t)\xi_{KZ})$. The factor $\kappa(t)$ depends mildly on the time of measurement and $\kappa(-t_{KZ}) \approx 1$. Once again, this result is roughly that of a stationary system with correlation length ξ_{KZ}, as would be expected from Kibble–Zurek considerations. The factor $\kappa(t)$ is a correction to the extreme adiabatic–diabatic assumption. The paper [21] investigates an intermediate regime of fast quench (as described above). While this paper investigates scaling of S_{EE} as a function of quench rate in the slow regime, there is no similar analysis in the fast regime.

In this letter, we study entanglement entropy for a simple system: an infinite harmonic chain (i.e. a $1+1$ dimensional bosonic theory on a lattice) with a time dependent mass term which asymptotes to constant *finite* values at early and late times. We choose a mass function for which the quantum dynamics can be solved exactly. The use of such a protocol allows us to explore the whole range of quench rates, where the speed of quench is measured in units of the initial gap rather than the lattice scale. We compute the entanglement entropy for a subsystem of size l (in lattice units) in the middle of the quench and find that it scales in interesting ways as we change the quench rate. The dimensionless quantity which measures the quench timescale is $\Gamma_Q = m_0\delta t$ where m_0 is the initial gap.

2. Our setup and quench protocols

The hamiltonian of the harmonic chain is given by

$$H = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left[P_n^2 + (X_{n+1}-X_n)^2 + m^2(t)X_n^2\right] \tag{3}$$

where (X_n, P_n) are the usual canonically conjugate scalar field variables on an one dimensional lattice whose sites are labelled by the integer n. The mass term $m(t)$ is time dependent. All quantities are in lattice units. In terms of momentum variables X_k, P_k

$$X_n(t) = \int_{-\pi}^{\pi}\frac{dk}{2\pi}X_k(t)\,e^{ikn} \qquad P_n(t) = \int_{-\pi}^{\pi}\frac{dk}{2\pi}P_k(t)\,e^{ikn} \tag{4}$$

the equation of motion is given by

$$\frac{d^2X_k}{dt^2} + [4\sin^2(k/2) + m^2(t)]X_k = 0 \tag{5}$$

We are interested in functions $m(t)$ which asymptote to constant values m_0 at $t \to \pm\infty$, and pass through zero at $t=0$. Let $f_k(t)$ be a solution of (5) which asymptotes to a purely positive frequency solution $\sim e^{-i\omega_0 t}/\sqrt{2\omega_0}$ at $t \to -\infty$, where

$$\omega_0^2 = 4\sin^2(k/2) + m_0^2. \tag{6}$$

A mode decomposition

$$X_k(t) = f_k(t)a_k + f_k^\star(t)a_{-k}^\dagger \tag{7}$$

with $[a_k, a_{-k'}^\dagger] = 2\pi\delta(k-k')$ can be then used to define the "in" vacuum by $a_k|0\rangle = 0$ for all k. The solutions $f_k(t)$ are chosen to satisfy the Wronskian condition $f_k(\dot{f}_k)^\star - (\dot{f}_k)f_k^\star = i$. The state $|0\rangle$ then denotes the Heisenberg picture ground state of the initial Hamiltonian. The normalized wavefunctional for the "in" vacuum state is given by

$$\Psi_0(X_k,t) = \prod_k \frac{1}{[\sqrt{2\pi}\,f_k^\star(t)]^{1/2}}\exp\left[\frac{1}{2}\left(\frac{\dot{f}_k(t)}{f_k(t)}\right)^\star X_kX_{-k}\right] \tag{8}$$

We will choose a quench protocol for a mass function for which the mode functions $f_k(t)$ can be solved exactly. The particular mass function we use is

$$m^2(t) = m_0^2\tanh^2(t/\delta t) \tag{9}$$

The corresponding mode functions are given by

$$f_k = \frac{1}{\sqrt{2\omega_0}}\frac{(2)^{i\omega_0\delta t}\cosh^{2\alpha}(t/\delta t)}{E'_{1/2}\tilde{E}_{3/2} - E_{1/2}\tilde{E}'_{3/2}} \times$$
$$[\tilde{E}'_{3/2}{}_2F_1(\tilde{a},\tilde{b},\frac{1}{2};-\sinh^2(t/\delta t))$$
$$+E'_{1/2}\sinh(t/\delta t){}_2F_1(\tilde{a}+\frac{1}{2},\tilde{b}+\frac{1}{2},\frac{3}{2};-\sinh^2(t/\delta t))]$$

where we have defined

$$\begin{aligned}
\omega_0^2(k) &= 4\sin^2(k/2) + m_0^2\\
\alpha &= \frac{1}{4}[1+\sqrt{1-4m_0^2\delta t^2}]\\
\tilde{a} &= \frac{1}{4}[1+\sqrt{1-4m_0^2\delta t^2}] + \frac{i}{2}\delta t\omega_0\\
\tilde{b} &= \frac{1}{4}[1+\sqrt{1-4m_0^2\delta t^2}] - \frac{i}{2}\delta t\omega_0\\
E_{1/2} &= \frac{\Gamma(1/2)\Gamma(\tilde{b}-\tilde{a})}{\Gamma(\tilde{b})\Gamma(1/2-\tilde{a})}\\
\tilde{E}_{3/2} &= \frac{\Gamma(3/2)\Gamma(\tilde{b}-\tilde{a})}{\Gamma(\tilde{b}+1/2)\Gamma(1-\tilde{a})}\\
E'_c &= E_c(\tilde{a}\leftrightarrow\tilde{b}).
\end{aligned} \tag{10}$$