Physics Letters B 772 (2017) 159–163

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Deuteron properties from muonic atom spectroscopy

N.G. Kelkar *, D. Bedoya Fierro

Dept. de Fisica, Universidad de los Andes, Cra. 1E No. 18A-10, Santafe de Bogotá, Colombia

ARTICLE INFO

Article history:
Received 18 November 2016
Received in revised form 10 June 2017
Accepted 14 June 2017
Available online 19 June 2017
Editor: W. Haxton

Keywords:
Deuteron radius
Electromagnetic form factors
Muonic atoms

ABSTRACT

Leading order (α^4) finite size corrections in muonic deuterium are evaluated within a few body formalism for the $\mu^- pn$ system in muonic deuterium and found to be sensitive to the input of the deuteron wave function. We show that this sensitivity, taken along with the precise deuteron charge radius determined from muonic atom spectroscopy can be used to determine the elusive deuteron D-state probability, P_D, for a given model of the nucleon–nucleon (NN) potential. The radius calculated with a P_D of 4.3% in the chiral NN models and about 5.7% in the high precision NN potentials is favoured most by the $\mu^- d$ data.

1. Introduction

The lightest nucleus, namely, the deuteron, has traditionally held an important place in nuclear physics as a testing ground for the nucleon–nucleon interaction. Determining the D-state probability in the deuteron wave function in particular has been a classic problem of nuclear physics [1–3]. Stating the problem in simple words, the deuteron has a quadrupole moment and hence cannot be in a pure S-state but rather a D-state admixture is required. However, as it was shown in [4] that the D-state probability, $P_D = \int_0^\infty w^2(r)dr$ (with $w(r)$ being the deuteron radial wave function with $l = 2$), is inaccessible directly to experiments, it is usually the asymptotic D-state to S-state wave function ratio, η [2, 5], which is determined. There do exist attempts to determine P_D from the measured magnetic moment of the deuteron, μ_D, with, $\mu_D = \mu_S - (3/2)P_D(\mu_S - 1/2) + \delta_R$, where, $\mu_S = \mu_P + \mu_N$ is the isoscalar nucleon magnetic moment. However, the term δ_R which includes mesonic exchange effects, relativistic corrections, dynamical effects and isobar configurations in the deuteron introduces uncertainties in the extraction of P_D [6]. This fact was noticed in one of the oldest works by Feshbach and Schwinger [1] on the theory of nuclear forces which gave the D-state probability, P_D, ranging between 2% to 6%. Much later, Ref. [7] listed values of P_D ranging from 0.28 to 6.47% for 9 different nucleon–nucleon (NN) potentials. However, earlier in [8] the possible minimum was shown to be 0.45%. With P_D not being a measurable quantity, Refs. [2] and [5] determined the asymptotic ratio $\eta = 0.0256 \pm 0.0004$ and 0.0268 $\pm$ 0.0013 from tensor analyzing powers in sub-Coulomb (d, p) reactions and dp elastic scattering respectively. In the absence of a "measured" D-state probability, theoretical models of the NN interaction also try to reproduce the asymptotic ratio η determined from experiments (in addition to other data) to confirm the reliability of the NN model [3].

The purpose of this work is to present a new method which provides a means to fix the percentage of the "elusive" [4] D-state probability, P_D, from experiments in an indirect manner. The method is particularly useful in view of the very high precision reported by recent muonic deuterium experiments [9]. It is based on a few body calculation of the leading order (α^4) finite size corrections (FSC) to the energy levels of muonic deuterium atoms. There exists extensive literature on corrections including the deuteron polarization [10–12], with detailed calculations of FSC at higher orders (α^5, α^6 etc) [13,10–12]. The sensitivity of the higher order FSC to the form of the nucleon–nucleon potential (and hence the deuteron wave function) is found to be small [10,12] or negligible [14]. The leading FSC at order α^4 in these works is written in terms of the deuteron charge radius. The few body formalism of the present work helps in revealing the dependence of the leading FSC term on the proton and neutron form factors as well as the deuteron wave function. We show that a comparison of the order α^4 FSC with those of Ref. [9] where the radius is precisely extracted from measurements in muonic deuterium provides a method to adjust the deuteron D-state probability. To be specific, we present calculations using different parametrizations of the deuteron wave function (with different amounts of the D-state probabilities) and compare the corrections with those given in [9] in a form dependent on the deuteron charge radius, r_d. Though

* Corresponding author.
E-mail address: nkelkar@uniandes.edu.co (N.G. Kelkar).

http://dx.doi.org/10.1016/j.physletb.2017.06.042

the general trend of the results is an increase in the radius for smaller values of P_D, the results are found to depend on the type of model used. In the class of chiral models [15], $P_D = 4.3\%$ is found to be favourable for the closest agreement with the precise value of $r_d = 2.12562(78)$ fm [9]. Using high precision NN potentials such as Nijmegen, Reid, Paris etc. [16], $P_D = 5.7\%$ to 5.8% is favoured by the μd data.

2. Finite size effects in muonic deuterium

Finite size corrections (FSC) to the energy levels in the hydrogen atom has been a topic of revived interest [17] in the past few years due to the increase in the precision achieved in atomic spectroscopy measurements. These effects are manifested more strongly in muonic atoms due to the fact that the muon is about 200 times heavier than the electron and hence has a Bohr radius which is much smaller. In view of the recent precise measurement of the Lamb shift in muonic deuterium [9], it seems timely to put forth the question as to what other impact (apart from the precise radius determination) does this measurement have on physics. In order to see this, we study the effects of deuteron structure on the energy levels in this atom. The present work considers the effects at leading order (α^4) and we refer the reader to [10–12] for higher order corrections.

2.1. Electromagnetic muon–deuteron potential

We investigate the finite size effects by calculating the energy correction, ΔE, using first order perturbation theory involving an electromagnetic muon–deuteron potential, $V_{\mu\text{-}d}$. The latter is constructed using a three body approach to the muon–proton–neutron system with the proton and neutron being bound inside the deuteron. As we will see below, the $\mu^- p$ and $\mu^- n$ interactions are obtained using the proton and neutron electromagnetic form factors and the pn interaction is contained in the deuteron wave function. Such a potential can be constructed using standard techniques from scattering theory where we first write down the scattering amplitude to obtain the potential $V_{\mu\text{-}d}(\boldsymbol{q})$ in momentum space and then evaluate its Fourier transform which enters the energy correction given by, $\Delta E = \int_0^\infty \Delta V(r) |\Psi_{nl}(\boldsymbol{r})|^2 d^3r$. This procedure of obtaining potentials in coordinate space is also common in quantum field theory [18–20]. Here, ΔV is the difference of $V_{\mu\text{-}d}(r)$ and the $\mu^- d$ electromagnetic potential assuming the deuteron to be point-like. Details of the few body formalism used here can be found in [21,22]. We shall repeat the relevant steps briefly below.

The Hamiltonian of the quantum system consisting of a muon and a nucleus (with A nucleons) is given as [21], $H = H_0 + V_{\mu\text{-}A} + H_A$, where H_0 is the muon-nucleus kinetic energy operator (free Hamiltonian), $V_{\mu\text{-}A} = \sum_{i=1}^{A} V_i$, the sum of muon–nucleon potentials, $V_i \equiv V_{\mu\text{-}N}(|\boldsymbol{R} - \boldsymbol{r}_i|)$, where $\boldsymbol{R}$ and $\boldsymbol{r}_i$ are the coordinates of the muon and the ith nucleon with respect to the centre of mass of the nucleus and H_A is the total Hamiltonian of the nucleus containing the potential term, $\sum_{i\neq j} V_{NN}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$. We proceed with the assumption that the nucleus remains in its ground state during the scattering process, i.e., $H_A |\Phi\rangle = \epsilon |\Phi\rangle$ and that the nucleons occupy fixed positions inside the nucleus. The muon-nucleus elastic scattering amplitude can be expressed as [21] $f(\boldsymbol{k}', \boldsymbol{k}; E) = -(\mu/\pi) \langle \boldsymbol{k}', \Phi | T(E) | \boldsymbol{k}, \Phi\rangle$ in terms of the matrix elements of the operator T obeying the Lippmann–Schwinger (L-S) equation, $T = V + V(E - H_0 - H_A)^{-1} T$. $|\boldsymbol{k}, \Phi\rangle$ and $|\boldsymbol{k}', \Phi\rangle$ are the initial and final asymptotic states which differ only in the direction of the relative muon nucleus momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$. Since the electromagnetic potential, V_{μ^-A}, is proportional to the coupling constant $\alpha \sim 1/137$, it is reasonable to truncate the L-S equation at first order and approximate $T = V = \sum_i V_i$. Thus, $T(\boldsymbol{k}', \boldsymbol{k}) = V(\boldsymbol{k}', \boldsymbol{k})$ and denoting, $T(\boldsymbol{k}', \boldsymbol{k}) \equiv \langle \boldsymbol{k}', \Phi | T(E) | \boldsymbol{k}, \Phi\rangle$, we have $V(\boldsymbol{k}', \boldsymbol{k}) = \langle \boldsymbol{k}', \Phi | \sum_{i=1}^{A} V_i | \boldsymbol{k}, \Phi\rangle$. If the internal Jacobi coordinates are denoted by $\boldsymbol{x}_i$, then relating them with $\boldsymbol{r}_i = a_i \boldsymbol{x}_1 + b_i \boldsymbol{x}_2 + \ldots + g_i \boldsymbol{x}_{A-1}$, we can write,

$$V(\boldsymbol{k}', \boldsymbol{k}) = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2 \ldots d\boldsymbol{x}_{A-1}\, |\Phi(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots)|^2 \sum_{i=1}^{A} V_i(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{r}_i)\,, \tag{1}$$

where, $V_i(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{r}_i) = V_i(\boldsymbol{k}', \boldsymbol{k}) \exp[i(\boldsymbol{k} - \boldsymbol{k}') \cdot \boldsymbol{r}_i]$. The above discussion is valid for any nucleus with A nucleons. In case of the muon–deuteron system, this reduces to

$$V(\boldsymbol{k}', \boldsymbol{k}) = \int d\boldsymbol{x}_1\, |\Phi_d(\boldsymbol{x}_1)|^2 \left[\, V_{\mu\text{-}p}(\boldsymbol{k}', \boldsymbol{k}, \frac{1}{2}\boldsymbol{x}_1) + V_{\mu\text{-}n}(\boldsymbol{k}', \boldsymbol{k}, -\frac{1}{2}\boldsymbol{x}_1) \,\right] \tag{2}$$

where we used, $\boldsymbol{x}_1 = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $\boldsymbol{r}_1 = (1/2)\boldsymbol{x}_1$ and $\boldsymbol{r}_2 = -(1/2)\boldsymbol{x}_1$. We identify 1 and 2 with proton and neutron so that, $V_1 = V_{\mu^- p}$, $V_2 = V_{\mu^- n}$ and Φ_d is the deuteron wave function.

To evaluate (2), we need the μ^--nucleon electromagnetic potential, which, with the inclusion of the nucleon electromagnetic form factors $G_E^N(q^2)$ can be written using the formalism of the Breit equation [18] within the one-photon-exchange interaction. Since such a potential was explicitly derived in [17,18] by evaluating the elastic muon–nucleon amplitude expanded in powers of $1/c^2$, we shall not repeat the derivation here. This potential with form factors contains 23 terms [18] corresponding to the (i) Coulomb potential, (ii) Darwin terms, and (iii) spin dependent terms which give rise to fine and hyperfine structure. If we consider only the scalar parts of the Breit potential, they depend only on $\boldsymbol{q}^2$ and hence we can write, $V_{\mu\text{-}N}(\boldsymbol{k}, \boldsymbol{k}') \equiv V_{\mu\text{-}N}(\boldsymbol{q})$ [18,17], where, $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}'$ is the momentum transfer carried by the exchanged photon. Denoting $Q = |\boldsymbol{q}|$, the $\mu^- N$ potential is given as [17],

$$V_{\mu^- N}(Q) = -4\pi\alpha \frac{G_E^N(Q^2)}{Q^2} \left\{ 1 - \frac{Q^2}{8m_N^2 c^2} - \frac{Q^2}{8m_\mu^2 c^2} \right\}, \tag{3}$$

where m_N and m_μ are the nucleon and muon masses. $G_E^N(Q^2)$ is the nucleon electric form factor. A Fourier transform of the first term in the curly bracket leads to the $\mu^- N$ Coulomb potential for a finite sized nucleon. The next two terms in the curly brackets are relativistic corrections, the Darwin terms in the muon (spin 1/2)–nucleon (spin 1/2) $\mu^- N$ interaction Breit potential. The Darwin term $Q^2/8m_N^2c^2$ is conventionally not considered as a part of the nucleon form factor $G_E^N(q^2)$ [23] and hence is kept explicitly in the muon–nucleon potential here. Putting together (2) and (3) we obtain the muon–deuteron electromagnetic potential, $V_{\mu\text{-}d}(Q) = V_{\mu\text{-}p}(Q) \int d\boldsymbol{x}\, |\Phi_d(\boldsymbol{x})|^2 e^{-i\boldsymbol{q}\cdot\boldsymbol{x}/2} + V_{\mu\text{-}n}(Q) \int d\boldsymbol{x}\, |\Phi_d(\boldsymbol{x})|^2 e^{i\boldsymbol{q}\cdot\boldsymbol{x}/2}$, in momentum space. The integrals in this expression can be shown to reduce to [7] $G_0(Q) = \int_0^\infty [u^2(r) + w^2(r)]\, j_0(Qr/2)\, dr$, where, $u(r)$ and $w(r)$ are the radial parts of the deuteron S- and D-wave functions. Thus, $V_{\mu\text{-}d}(Q) = (V_{\mu\text{-}p}(Q) + V_{\mu\text{-}n}(Q)) G_0(Q)$, so that,

$$V_{\mu\text{-}d}(Q) = -4\pi\alpha \frac{G_0(Q)[G_E^p(Q^2) + G_E^n(Q^2)]}{Q^2} \left(1 - \frac{Q^2}{8m_N^2} - \frac{Q^2}{8m_\mu^2} \right), \tag{4}$$

where the proton and neutron masses have been written as $m_p \approx m_n \approx m_N$ for simplicity. We note here that the three body formalism allows us to include the relativistic corrections in the form of