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On the stability of non-supersymmetric supergravity solutions

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A R T I C L E I N F O

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A B S T R A C T

We examine the stability of some non-supersymmetric supergravity solutions that have been found recently. The first solution is $AdS_5 \times M_6$, for M_6 an stretched CP^3. We consider breathing and squashing mode deformations of the metric, and find that the solution is stable against small fluctuations of this kind. Next we consider type IIB solution of $AdS_2 \times M_8$, where the compact space is a $U(1)$ bundle over $N(1,1)$. We study its stability under the deformation of M_8 and the 5-form flux. In this case we also find that the solution is stable under small fluctuation modes of the corresponding deformations.

1. Introduction

Selecting a stable solution among the many candidate supergravity solutions is a major problem in any Kaluza–Klein compactification. One way to guarantee the stability is to demand that the solution preserve a portion of supersymmetry [1–3]. In the absence of supersymmetry, on the other hand, it is difficult to conclude whether a particular solution is stable. In fact, one needs to examine the stability under small perturbations in all possible directions of the potential. Moreover, even if a solution is stable under such small perturbations, there still remains the question of stability under nonperturbative effects [4]. Finding non-supersymmetric stable solutions, however, becomes important if we are to construct realistic phenomenological models in which supersymmetry is spontaneously broken.

Freund–Rubin solutions can be divided into two main classes depending on whether or not the compact space encompasses (electric) fluxes [5,6]. When the flux has components only along the AdS direction, it has been observed that the majority of solutions either preserve supersymmetry (and hence stable), or at least are perturbatively stable. For solutions that support flux in the compact direction (Englert type), however, supersymmetry is often broken. They are in fact suspected to be unstable, though, the direct computation of mass spectrum and determination of stability is more involved. Englert type solution of $AdS_4 \times S^7$, for instance, was shown to be unstable [7], and this was further generalized to seven dimensional spaces which admit at least two Killing spinors [8]. Pope–Warner solution is another non-supersymmetric example which supports flux in the compact direction [9], and was proved to be unstable much later [10]. Englert type solutions, in spite of their possible instability, have played a key role in studying the holographic superconductors. By employing similar techniques that we use in this paper, domain wall solutions were found that interpolate between the Englert type and the skew-whiffed solutions. The domain wall solutions were then used to describe holographic superconductor phase diagrams [11].

The stability of Freund–Rubin type geometries of the form $AdS_p \times M_q$, where AdS_p is anti-de Sitter spacetime and M_q a compact manifold, has also increasingly been studied after the discovery of the AdS/CFT correspondence [12]. Stability is important for understanding a possible dual conformal field theory (CFT) description. For stable solutions, the spectrum of the masses directly yields the dimensions of certain operators in such a CFT. Unstable solutions can still have a dual CFT description but the physics is different [13]. Since the curvature of AdS is negative, not all the tachyonic modes lead to instability. In fact, scalars with $m^2 < 0$ may also appear if their masses are not below a bound set by the curvature scale of AdS [3].

Recently, some new non-supersymmetric compactifying solutions of eleven-dimensional supergravity and type IIB supergravity have been found [14,15]. Specifically, the eleven-dimensional supergravity solution consists of $AdS_5 \times M_6$, where for M_6 there are two possible choices. For the first solution M_6 is CP^3 with the standard Fubini-Study metric, which was derived and studied in [16], and it was further shown that is perturbatively stable [17]. For the second solution S^2 fibers of CP^3 are slightly stretched with respect to the base manifold. Type IIB solution, on the other hand, is $AdS_2 \times M_8$, where M_8 is a $U(1)$ bundle over $N(1,1)$. All these

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solutions have fluxes in the compact direction, they break supersymmetry and therefore it is important to know whether they are stable.

It is also interesting to see how these new solutions might arise from near horizon geometries of some particular brane configurations. This would then lead us to the construction of the CFT duals [18]. For the eleven-dimensional supergravity solutions, first notice that the compact manifold admits a nontrivial 2-cycle over which we can wrap branes. Therefore, one way to get the AdS_5 factor is to construct a Ricci flat cone over the compact manifold and then consider fractional 3-branes (wrapped M5-branes over the 2-cycles) in the orthogonal directions placed at the tip of the cone. The near horizon geometry of this brane configuration would be $AdS_5 \times M_6$. Similarly, for the type IIB case, since M_8 is Einstein and admits nontrivial 3-cycles, we can construct a Ricci flat cone over it, and then put fractional D0-branes (D3-branes wrapped over 3-cycles of M_8) at the tip of the cone. Therefore we expect $AdS_2 \times M_8$ solution to arise as the near horizon limit of this D0-brane configuration.

In this paper we examine the stability of solutions under small perturbations of the metric. For getting consistent equations of motion on AdS, however, we also need to introduce deformations of the fluxes. Here we follow an approach which is close to that of [19,20]. For compactification to AdS_5 the metric deformations correspond to the breathing and squashing modes. Including the deformation of the 4-form flux would correspond to three massive mode excitations on the AdS space. In type IIB case, however, the bundle structure of the compact manifold allows a more general deformation, which, in turn, results in seven massive mode excitations. Apart from deriving the mass spectrum of small fluctuations, our approach has the advantage of providing us with a set of consistent reduced equations on AdS space, so that any solution to these equations can be uplifted to a supergravity solution in eleven or ten dimensions.

2. Stability of $AdS_5 \times CP^3$ compactification

In this section we consider the solution $AdS_5 \times M_6$, where M_6 is CP^3 written as an S^2 bundle over S^4 [14], and study its stability under small perturbations. We start by deforming the metric along the fiber and the base by some unknown scalar functions on AdS_5. To get consistent reduced equations we see that the 4-form flux also needs to be deformed. After deriving the curvature tensor of the metric we write the supergravity equations of motion, and then linearize the equations around the known solutions. This allows us to read the mass of the small fluctuations corresponding to those deformations. If the mass squared falls in the Breitenlohner–Freedman range then the solution is stable against such perturbations.

To begin with, let us take the eleven dimensional spacetime to be the direct product of a 5 and 6-dimensional spaces,

$$ds_{11}^2 = ds_{AdS_5}^2 + ds_6^2. \tag{1}$$

For the 6-dimensional space the metric reads

$$ds_6^2 = d\mu^2 + \frac{1}{4}\sin^2\mu\,\Sigma_i^2 + \lambda^2(d\theta - \sin\phi A_1 + \cos\phi A_2)^2 + \lambda^2\sin^2\theta\big(d\phi - \cot\theta(\cos\phi A_1 + \sin\phi A_2) + A_3\big)^2, \tag{2}$$

with λ the squashing parameter, and

$$A_i = \cos^2\frac{\mu}{2}\Sigma_i, \tag{3}$$

$$d\Sigma_i = -\frac{1}{2}\epsilon_{ijk}\Sigma_j \wedge \Sigma_k. \tag{4}$$

This is an S^2 bundle over S^4, and for $\lambda^2 = 1$ we get the Fubini–Study metric on CP^3.

To discuss the stability, we deform the metric as follows:

$$\begin{aligned} d\bar{s}^2 &= e^{2A(x)}g_{\alpha\beta}dx^\alpha dx^\beta \\ &+ e^{2B(x)}\left(d\mu^2 + \frac{1}{4}\sin^2\mu\,\Sigma_j^2\right) \\ &+ e^{2C(x)}(d\theta - \sin\phi A_1 + \cos\phi A_2)^2 \\ &+ e^{2C(x)}\sin^2\theta\big(d\phi - \cot\theta(\cos\phi A_1 + \sin\phi A_2) + A_3\big)^2, \end{aligned} \tag{5}$$

where $g_{\alpha\beta}$ is the AdS_5 metric, and $A(x)$, $B(x)$, and $C(x)$ are arbitrary scalar functions on AdS_5. In fact, $B(x)$ and $C(x)$ correspond to what is usually called the breathing and the squashing mode deformations. We choose the following vielbein basis

$$\begin{aligned} \bar{e}^\alpha &= e^{A(x)}e^\alpha \qquad \alpha = \bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4} \\ \bar{e}^0 &= e^{B(x)}e^0 \\ \bar{e}^i &= e^{B(x)}e^i \qquad i = 1, 2, 3 \\ \bar{e}^a &= e^{C(x)}e^a \qquad a = 5, 6, \end{aligned} \tag{6}$$

where the indices $\alpha, \beta, \dots$ indicate the 5d spacetime coordinates, and the rest are related to the 6-dimensional space, and

$$\begin{aligned} e^0 &= d\mu, \qquad e^i = \frac{1}{2}\sin\mu\,\Sigma_i, \\ e^5 &= \lambda(d\theta - \sin\phi A_1 + \cos\phi A_2), \\ e^6 &= \lambda\sin\theta\big(d\phi - \cot\theta(\cos\phi A_1 + \sin\phi A_2) + A_3\big). \end{aligned} \tag{7}$$

Evaluation of the Ricci tensor of this deformed metric yields

$$\begin{aligned} \bar{R}_{\alpha\beta} &= e^{-2A}\{R_{\alpha\beta} - \nabla^2 A\delta_{\alpha\beta} + 4\partial_\beta B\partial_\alpha(A - B) \\ &+ 2\partial_\beta C\partial_\alpha(A - C)\}, \end{aligned} \tag{8}$$

$$\bar{R}_{ij} = \big(3e^{-2B} - e^{2(C-2B)} - e^{-2A}\nabla^2 B\big)\delta_{ij}, \tag{9}$$

$$\bar{R}_{ab} = \big(e^{-2C} + e^{2(C-2B)} - e^{-2A}\nabla^2 C\big)\delta_{ab}. \tag{10}$$

Next, as in [14], we want to write a similar ansatz for the gauge field strength. However, since we have perturbed the metric with some scalar functions on AdS space we must add an extra term for consistency. Further, it is easier first to write the Hodge dual ansatz as follows

$$\bar{*}_{11}F_4 = \bar{\epsilon}_5 \wedge \big(\alpha(x)e^{56} + \gamma(x)K\big) + \bar{*}_5 d\eta \wedge Im\Omega, \tag{11}$$

where we have defined,

$$R_1 = \sin\phi\big(e^{01} + e^{23}\big) - \cos\phi\big(e^{02} + e^{31}\big), \tag{12}$$

$$\begin{aligned} R_2 &= \cos\theta\cos\phi\big(e^{01} + e^{23}\big) \\ &+ \cos\theta\sin\phi\big(e^{02} + e^{31}\big) - \sin\theta\big(e^{03} + e^{12}\big), \end{aligned} \tag{13}$$

$$\begin{aligned} K &= \sin\theta\cos\phi\big(e^{01} + e^{23}\big) + \sin\theta\sin\phi\big(e^{02} + e^{31}\big) \\ &+ \cos\theta\big(e^{03} + e^{12}\big), \end{aligned} \tag{14}$$

$$Re\Omega = R_1 \wedge e^5 + R_2 \wedge e^6, \tag{15}$$

$$Im\Omega = R_1 \wedge e^6 - R_2 \wedge e^5, \tag{16}$$

$$\omega_4 = e^0 \wedge e^1 \wedge e^2 \wedge e^3. \tag{17}$$

As $F_4 \wedge F_4 = 0$ (see (21)), the Maxwell equation reads

$$d\bar{*}_{11}F_4 = \bar{\epsilon}_5 \wedge (\alpha - \gamma) \wedge Im\Omega + d\bar{*}_5 d\eta \wedge Im\Omega = 0, \tag{18}$$

where we used [14],