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Cosmic information, the cosmological constant and the amplitude of primordial perturbations

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ABSTRACT

A unique feature of gravity is its ability to control the information accessible to any specific observer. We quantify the notion of cosmic information ('CosmIn') for an eternal observer in the universe. Demanding the finiteness of CosmIn *requires* the universe to have a late-time accelerated expansion. Combining the introduction of CosmIn with generic features of the quantum structure of spacetime (e.g., the holographic principle), we present a holistic model for cosmology. We show that (i) the numerical value of the cosmological constant, as well as (ii) the amplitude of the primordial, scale invariant, perturbation spectrum can be determined in terms of a single free parameter, which specifies the energy scale at which the universe makes a transition from a pre-geometric phase to the classical phase. For a specific value of the parameter, we obtain the correct results for both (i) and (ii). This formalism also shows that the quantum gravitational information content of spacetime can be tested using precision cosmology.

It is now well established that information is a physical entity [1] and the flow of information has concrete physical consequences. The fact that gravity controls the amount of spacetime information accessible to a given observer, suggests that one can acquire deeper insights into spacetime dynamics through its information content. The concept of information, being a common ingredient in both classical and quantum regimes, can thus be used to provide a link between the descriptions of spacetime in these two domains.

The key difficulty in formulating this connection lies in quantifying the amount of spacetime information. While this is indeed difficult for a *general* spacetime, we show that it is possible to introduce a natural definition of information content in the context of *cosmological* spacetimes ('CosmIn') and use it to link the quantum and classical phases of the universe. Moreover, we shall see that this information paradigm allows us to determine both, (i) the numerical value of the cosmological constant and (ii) the amplitude of the primordial, scale invariant, power spectrum of perturbations, thus providing a holistic description of cosmology.

In any Friedmann model, the *proper* length-scales (say, the wavelengths of the modes of a field) scale as $\lambda(a) \propto a$ and can cross the *proper* Hubble radius $H^{-1}(a) = (\dot{a}/a)^{-1}$ as the universe evolves. The number of modes dN located in the comoving Hubble volume $V_H(a) = (4\pi/3)(aH)^{-3}$, which have comoving wave numbers in the range d^3k, is given by $dN = V_H(a)d^3k/(2\pi)^3 \equiv V_H(a)dV_k/(2\pi)^3$ where $dV_k = 4\pi k^2 dk$. A mode with a comoving wave number k crosses the Hubble radius when $k = k(a) \equiv aH(a)$. So, the modes with wave numbers between k and $k + dk$, where $dk = [d(aH)/da]\, da$, cross the Hubble radius during the interval $(a, a + da)$. We *define* the information associated with modes which cross the Hubble radius during any interval $a_1 < a < a_2$ by

$$N(a_2, a_1) = \pm \int_{a_1}^{a_2} \frac{V_H(a)}{(2\pi)^3} \frac{dV_k[k(a)]}{da}\, da = \pm \frac{2}{3\pi} \ln\left(\frac{h_1}{h_2}\right) \qquad (1)$$

where $h(a) \equiv H^{-1}(a)/a$ is the *comoving* Hubble radius and $h_1 = h(a_1), h_2 = h(a_2)$. The sign is chosen to keep N positive, by definition.

In the absence of any untested physics from the matter sector (like e.g., inflationary scalar fields, which we will *not* invoke in this paper), the universe is radiation dominated at early epochs and, classically, has a singularity at $a = 0$. In reality, the classical description breaks down when quantum gravitational effects set in. We assume that the universe makes a transition from a quantum, pre-geometric phase to the classical, geometric phase at an epoch $a = a_{QG}$ when the radiation energy density is $\rho_R = \rho_{QG}$

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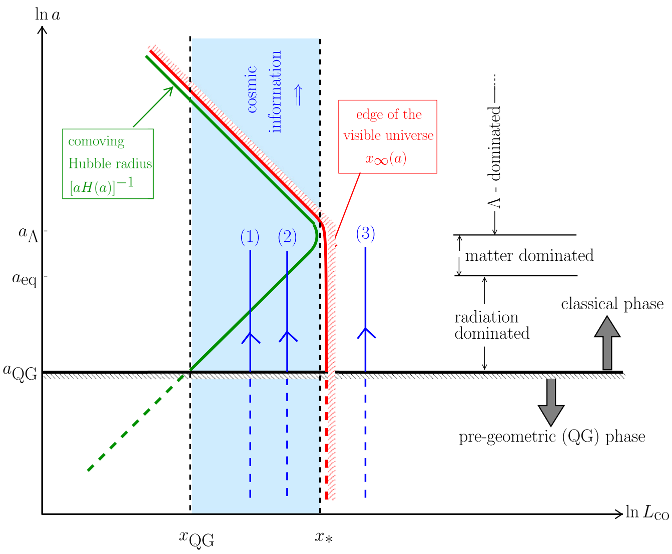

Fig. 1. Various length scales of interest in cosmological evolution. See text for discussion. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

where $(8\pi/3)\rho_{\rm QG} \equiv E_{\rm QG}^4$. We express the energy scale as $E_{\rm QG} \equiv \nu^{-1} E_{\rm Pl}$ where $E_{\rm Pl} \equiv \hbar c/L_{\rm P} = 1/L_{\rm P}$ in natural units ($\hbar = 1 = c$) and $L_P \equiv (G\hbar/c^3)^{1/2} = G^{1/2}$ is the Planck length; ν is a numerical factor which, as we shall see, can be determined from observations [2]. The Hubble radius at $a = a_{\rm QG}$ is $H_{\rm QG}^{-1} \equiv \nu^2 L_P$.

If the universe was populated by sources which satisfy $(\rho + 3p) > 0$ for all $a > a_{\rm QG}$, then the function $N(a, a_{\rm QG})$, defined by Eq. (1), is a monotonically increasing function of a and *diverges* as $a \to \infty$. It is reasonable to demand that $N(a, a_{\rm QG})$ *should be finite and its finite value should be determined by purely quantum gravitational (QG) considerations.* Such a behavior is a natural consequence of the discreteness of space and the existence of a minimal length — which is a generic feature of quantum gravity models — leading to a finite reservoir of information in the QG phase. This would require the comoving Hubble radius $H^{-1}(a)$ to reach a maximum value at some epoch, say, $a = a_\Lambda$ and then decrease. [If we do not invoke untested physics, and consider a universe made of radiation, matter and (possibly) the cosmological constant, then it is easy to show that N cannot be finite and monotonic, *asymptoting* to a constant. This can be seen from Fig. 1, in which each mode is represented by a vertical (blue) line. Unless the *comoving* Hubble radius asymptotes to a constant, more and more modes will keep entering the Hubble radius, and N cannot asymptotically approach a constant value. Clearly, the *comoving* Hubble radius cannot asymptote to a constant in a universe containing radiation, matter and possibly the cosmological constant.] Hence, the number of modes $N(a_\Lambda, a_{\rm QG})$ which *enter* the Hubble radius during the entire history of the universe — which we call 'CosmIn' — will be a finite constant, say $N(a_\Lambda, a_{\rm QG}) \equiv I_c$. This, in turn, requires $\rho + 3p = 0$ at $a = a_\Lambda$ with $\rho + 3p < 0$ for $a > a_\Lambda$. *The finiteness of CosmIn thus demands that we must have an accelerating phase in the universe.*

The simplest way to ensure that $(\rho + 3p) < 0$ at late times without invoking untested physics (like e.g., quintessence) is to introduce a non-zero cosmological constant, with energy density ρ_Λ. The expansion of such a universe, for $a > a_{\rm QG}$, is driven by the energy density of matter $\rho_m \propto a^{-3}$, radiation $\rho_R \propto a^{-4}$ and the cosmological constant ρ_Λ. Defining the density $\rho_{\rm eq} \equiv \rho_m^4(a)/\rho_R^3(a)$ which is a constant independent of a, we can model the universe as a dynamical system described by three densities: $(\rho_{\rm QG}, \rho_{\rm eq}, \rho_\Lambda)$.

Observations indicate that $\rho_{\rm eq} = [0.86 \pm 0.09\ {\rm eV}]^4$ and $\rho_\Lambda = [(2.26 \pm 0.05) \times 10^{-3}\ {\rm eV}]^4$. The theoretical status of these numerical values of $\rho_{\rm eq}$ and ρ_Λ are very different. The value of $\rho_{\rm eq}$ depends on the nature and abundance of dark matter and baryons relative to photons and — in principle — can be determined from high-energy physics. But, as is well-known, we do not have any theoretical basis to determine ρ_Λ which is considered a major challenge in theoretical physics.

However, in our approach, the value of ρ_Λ is determined by the value of $N(a_\Lambda, a_{\rm QG}) \equiv I_c$. The calculation of I_c is completely straightforward but a bit tedious. (See Appendix C of [3] for details.) The final result is given by:

$$I_c = -\frac{2}{3\pi} \ln\left[\frac{k_1(\rho_\Lambda^2 \rho_{\rm eq})^{1/12}}{E_{\rm QG}}\right] \tag{2}$$

where $k_1 = (3^{1/2}/2^{1/3})(8\pi/3)^{1/4} \approx 2.34$. Inverting this equation, we can express the cosmological constant in terms of $I_c, \nu, \rho_{\rm eq}$ as:

$$\rho_\Lambda L_P^4 = \frac{4}{27}\left(\frac{3}{8\pi}\right)^{3/2} \frac{1}{\nu^6(\rho_{\rm eq} L_P^4)^{1/2}} \exp\left(-9\pi I_c\right) \tag{3}$$

As claimed earlier, the *non-zero* value of the cosmological constant is related to the *finite* value of I_c. The fact that even an eternal observer can only access a finite amount of information (quantified in terms of the number of modes which cross the Hubble radius) implies that the cosmological constant is non-zero; we see that $\rho_\Lambda \to 0$ when $I_c \to \infty$ and vice-versa.

If I_c is known from an independent consideration, Eq. (3) will determine the numerical value of the cosmological constant in terms of $(\rho_{\rm eq}, \rho_{\rm QG})$. To have an independent handle on I_c, we consider some well-established results which are fairly independent of the choice of model of quantum gravity. One such result is that *the effective dimension of the quantum-corrected spacetime becomes $D = 2$ close to Planck scales,* independent of the original D. This result was obtained, in a fairly model-independent manner (using a renormalized quantum effective metric) in Ref. [4]. Similar results have been established earlier by several authors (for a sample, see e.g., [5]) in a number of approaches to quantum gravity. This, in turn, implies that [4,6] the unit of information associated with a quantum gravitational 2-sphere of radius L_P can be taken to be $I_{\rm QG} = 4\pi L_P^2/L_P^2 = 4\pi$. We shall therefore introduce the *postulate* that:

$$I_c = N(a_\Lambda, a_{\rm QG}) = 4\pi \tag{4}$$

We view this relation as a relic of the pre-geometric phase described by quantum gravitational considerations. While it suggests the notion of a "single Planckian sphere" from which the cosmogenesis started, it is *not* possible to model such an idea rigorously, given our current ignorance of quantum gravity and cosmogenesis. What is actually being postulated — *and what is sufficient for our purpose, independent of the details of the model* — is that the information content, as measured by CosmIn, $N(a_\Lambda, a_{\rm QG})$, is equal to the information contained in the two dimensional surface of a Planckian sphere, viz. 4π. As long as the cosmogenesis model maintains this equality of information, our results will follow. With this consideration, $I_c = 4\pi$ and we obtain

$$\rho_\Lambda L_P^4 = \frac{4}{27}\left(\frac{3}{8\pi}\right)^{3/2} \frac{1}{\nu^6(\rho_{\rm eq} L_P^4)^{1/2}} \exp\left(-36\pi^2\right) \tag{5}$$

Given the scale $E_{\rm QG} = \nu^{-1} E_P$ at which classical geometry arises from quantum pre-geometry, the above equation determines ρ_Λ. At this stage, we can also reverse the argument and use the observed value of ρ_Λ to determine the factor ν. Using the result $\rho_\Lambda L_P^4 = (1.14 \pm 0.09) \times 10^{-123}$ and $\rho_{\rm eq} L_P^4 = (2.41 \pm 1.01) \times 10^{-113}$, we find that $\nu = (6.2 \pm 0.3) \times 10^3$ making $E_{\rm QG}$ close to the GUTs