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Screening three-form fields

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A B S T R A C T

Screening mechanisms for a three-form field around a dense source such as the Sun are investigated. Working with the dual vector, we can obtain a thin-shell where field interactions are short range. The field outside the source adopts the configuration of a dipole which is a manifestly distinct behaviour from the one obtained with a scalar field or even a previously proposed vector field model. We identify the region of parameter space where this model satisfies present solar system tests.

1. Introduction

In the last one hundred years, the theory of General Relativity has given us a framework to explore Gravity from the orbits of planets to the evolution of the whole Universe. Nonetheless, a number of puzzling issues still remain, the most disconcerting one being the lack of explanation for the recent acceleration of the Universe. When seeking to extend General Relativity, theorists often invoke the presence of a dynamical scalar field either in the gravitational part of the action or instead, in the matter sector. This procedure is naturally dangerous as the field might couple to the rest of the world leading to fifth-forces and violating the equivalence principle. In order to avoid such constraints, screening mechanisms that impose a constant value of the field inside a body such as the Earth or the Sun, have been proposed. The way this is achieved consists in choosing a particular form for the potential and coupling of the field with baryons that ensures a large mass in the interior of the body, hence imposing short range interactions. In space, however, the field's mass is small and mediates an interaction of gravitational strength. These models are known by the name of chameleon mechanism [1–8]. An alternative set up is the symmetron [9], with a large vacuum expectation value in environments of low density and a small expectation value in environments of high density. As the coupling is proportional to the vacuum expectation value, the field decouples from the matter fields in regions of high density. Derivative couplings [10,11] have also been proposed as screening mechanisms. In the Vainshtein mechanism [12–15], the derivative self-couplings of the scalar field become large in the neighbourhood of a massive body. These non-linear contributions increase the kinetic terms of perturbations and suppress the strength of the interactions of the scalar field with matter.

Vector fields also exist in nature and one naturally wonders whether screening mechanisms also exist for those. This was investigated in Ref. [16] where it is shown that the mechanism is very similar to the symmetron and indeed such features exist with the additional effect that Lorentz invariance can also be shielded for a vector field. We can take this investigation one step further and study how a higher rank tensor behaves inside and in the neighbourhood of a compact object. In this article, we will focus on a vector dual to a three-form field [17–19]. The equations of motion are distinct from the ones of the vector field in Ref. [16] and consequently, the phenomenology obtained also differs considerably. Furthermore, we choose a form of the potential and coupling that results in a large vacuum expectation value of the field in high dense regions and conversely a small value in low dense regions. This allows us to recover homogeneity and isotropy on large scales. We first motivate the vector model from a three-form action and obtain the equations of motion. With our specific choice of potential and coupling, the field profile around a spherical source is computed and compared against previous scalar and vector field model solutions. Finally we place bounds on the model parameters from current observational limits.

2. Coupled three-form theory

We start from the action for a three-form field A minimally coupled to gravity with a conformally coupled matter Lagrangian density $\tilde{\mathcal{L}}_M$,

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$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{48}F^2-V(A^2)\right]+\int d^4x\tilde{\mathcal{L}}_m, \quad (1)$$

where g is the determinant of the metric $g^{\mu\nu}$, R is the Ricci scalar for the same metric, $F^2=F^{\alpha\beta\gamma\delta}F_{\alpha\beta\gamma\delta}$ where the four-form F is a generalized Faraday form given by $F_{\alpha\beta\gamma\delta}=4\nabla_{[\alpha}A_{\beta\gamma\delta]}$ and V is the potential for the three-form field A with $A^2=A^{\alpha\beta\gamma}A_{\alpha\beta\gamma}$. Throughout we use signature $(-,+,+,+)$.

We consider the matter sector to consist of a pressureless perfect fluid with energy density $\tilde{\rho}$. Its Lagrangian density $\tilde{\mathcal{L}}_m=\sqrt{-\tilde{g}}L_m$ depends on the metric $\tilde{g}^{\mu\nu}$ which is related to the gravity and three-form metric through a conformal transformation

$$\tilde{g}_{\mu\nu}=\Omega^2(A^2)g_{\mu\nu}, \quad (2)$$

where we assume the conformal factor Ω to be only a function of A^2.

Varying the action with respect to the three-form field yields the equation of motion

$$\nabla_\alpha F^{\alpha\beta\gamma\delta}=12\left(\frac{\partial V}{\partial A^2}+\rho\frac{\partial\Omega}{\partial A^2}\right)A^{\beta\gamma\delta}, \quad (3)$$

where we have defined the usual matter energy density in the Einstein frame as $\rho=\Omega^3\tilde{\rho}$.

We can understand this equation as being sourced by an effective potential $V_{\rm eff}(A^2)=V(A^2)+\rho\Omega(A^2)$ with an explicit dependence on the surrounding fluid energy density ρ.

Since $F^{\alpha\beta\gamma\delta}$ is antisymmetric, we also have the constraint equation

$$\nabla_\beta\nabla_\alpha F^{\alpha\beta\gamma\delta}=0, \quad (4)$$

for the four-form.

3. Dual vector field

For ease of calculation it is convenient to recast the theory using a vector field. For this purpose we introduce the Hodge dual forms of A and F, such that

$$B_\alpha=\frac{1}{3!}\epsilon_{\alpha\beta\gamma\delta}A^{\beta\gamma\delta}, \qquad \Phi=\frac{1}{4!}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta\gamma\delta}, \quad (5)$$

with the inverse relations

$$A^{\beta\gamma\delta}=\epsilon^{\beta\gamma\delta\alpha}B_\alpha, \qquad F^{\alpha\beta\gamma\delta}=-\epsilon^{\alpha\beta\gamma\delta}\Phi, \quad (6)$$

where ϵ is the fully antisymmetric Levi-Civita tensor.[1] It is easy to verify that $\Phi=\nabla^\alpha B_\alpha$, and also that

$$A^2=-6B^2, \qquad F^2=-24\Phi^2. \quad (7)$$

The equation of motion (3) can now be written as

$$\nabla_\alpha\Phi=-2\left(\frac{\partial V}{\partial B^2}+\rho\frac{\partial\Omega}{\partial B^2}\right)B_\alpha, \quad (8)$$

and the constraint equation (4) becomes

$$\nabla_\alpha\nabla_\beta\Phi-\nabla_\beta\nabla_\alpha\Phi=0. \quad (9)$$

Recalling the effective potential $V_{\rm eff}(B^2)=V(B^2)+\rho\Omega(B^2)$, the equation of motion (8) simply yields

$$\nabla_\alpha\Phi=-\frac{\partial V_{\rm eff}}{\partial B^\alpha}. \quad (10)$$

Using $\Phi=\nabla^\mu B_\mu$, we can write the equation of motion Eq. (10) in terms of the vector field only

[1] $\epsilon_{\alpha\beta\gamma\delta}=\sqrt{-g}\,\varepsilon_{\alpha\beta\gamma\delta}$, with ε the Levi-Civita symbol.

$$\nabla_\alpha\left(\nabla^\mu B_\mu\right)=-\frac{\partial V_{\rm eff}}{\partial B^\alpha}. \quad (11)$$

Note that this is fairly different from Ref. [16] where the vector field equation is given by $\Box B_\mu=-\frac{\partial V_{\rm eff}}{\partial B^\mu}$.

4. Screening mechanism

4.1. Critical points

We want to look at the field around massive astrophysical objects. We can model this using a pressureless spherically symmetric object with a homogeneous energy density ρ_c and radius r_c surrounded by a low and homogeneous energy density background ρ_b.

We are interested in static solutions in Minkowski space, therefore all our time derivatives are set to zero. From Eq. (8) we can see that, provided $\partial V_{\rm eff}/\partial B^2\neq 0$, the time component of B_μ has to be zero. Therefore our vector is always space-like with $B_\mu=(0,\vec{B})$ and $B^2\geq 0$. Also, the dual scalar field Φ is the divergence of the spatial vector, $\Phi=\vec{\nabla}\cdot\vec{B}$. Hence, we will only work in the spatial 3 dimensions in what follows.

The first derivative of the effective potential obtained from Eq. (8) is

$$\frac{\partial V_{\rm eff}}{\partial B^i}=2\left(\frac{\partial V}{\partial B^2}+\rho\frac{\partial\Omega}{\partial B^2}\right)B_i\,, \quad (12)$$

and the equation of motion for $\vec{B}$ is

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{B}\right)=-\frac{\partial V_{\rm eff}}{\partial\vec{B}}\,. \quad (13)$$

From Eq. (12) we find that a maximum of the effective potential can occur for $\vec{B}=0$ but also whenever the combination $\frac{\partial V}{\partial B^2}+\rho\frac{\partial\Omega}{\partial B^2}$ vanishes, yielding a broken $O(3)$ symmetry critical point. In either instance, the effective mass come from the second derivative of the effective potential,

$$\frac{\partial V_{\rm eff}}{\partial B^i\partial B^j}=2\left(\frac{\partial V}{\partial B^2}+\rho\frac{\partial\Omega}{\partial B^2}\right)\delta_{ij} +4\left(\frac{\partial^2 V}{(\partial B^2)^2}+\rho\frac{\partial^2\Omega}{(\partial B^2)^2}\right)B_iB_j\,. \quad (14)$$

As common practice in chameleon or symmetron screening mechanisms, the static profile is obtained by expanding the solutions at the maxima of the effective potential where the effective masses are negative. Therefore, at $\vec{B}=0$ we write the effective mass as

$$-m_0^2=2\left(\frac{\partial V}{\partial B^2}+\rho\frac{\partial\Omega}{\partial B^2}\right)\,. \quad (15)$$

When the symmetry is broken, we get a displaced critical point for the critical value $\vec{B}_c$ such that $\left.\frac{\partial V}{\partial B^2}+\rho\frac{\partial\Omega}{\partial B^2}\right|_{\vec{B}_c}=0$. At this point the field will get a mass in the $\vec{B}_c$ direction

$$-m_c^2=4\left.\left(\frac{\partial^2 V}{(\partial B^2)^2}+\rho\frac{\partial^2\Omega}{(\partial B^2)^2}\right)\right|_{\vec{B}_c}B_c^2\,, \quad (16)$$

whereas in the other two spatial orthogonal directions it will have an effective mass equal to zero.

From now on we will build our model with a specific form for our scalar potential

$$V(B^2)=-\frac{1}{2}m^2B^2-\frac{1}{4}B^4, \quad (17)$$

and for our conformal coupling