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Empirical information on nuclear matter fourth-order symmetry energy from an extended nuclear mass formula

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ABSTRACT

We establish a relation between the equation of state of nuclear matter and the fourth-order symmetry energy $a_{\mathrm{sym},4}(A)$ of finite nuclei in a semi-empirical nuclear mass formula by self-consistently considering the bulk, surface and Coulomb contributions to the nuclear mass. Such a relation allows us to extract information on nuclear matter fourth-order symmetry energy $E_{\mathrm{sym},4}(\rho_0)$ at normal nuclear density ρ_0 from analyzing nuclear mass data. Based on the recent precise extraction of $a_{\mathrm{sym},4}(A)$ via the double difference of the "experimental" symmetry energy extracted from nuclear masses, for the first time, we estimate a value of $E_{\mathrm{sym},4}(\rho_0) = 20.0 \pm 4.6$ MeV. Such a value of $E_{\mathrm{sym},4}(\rho_0)$ is significantly larger than the predictions from mean-field models and thus suggests the importance of considering the effects of beyond the mean-field approximation in nuclear matter calculations.

1. Introduction

The determination of the isospin dependent part of nuclear matter equation of state (EOS) has become a hot topic in both nuclear physics and astrophysics during the last decades [1–12]. The nuclear matter EOS tells us its energy per nucleon $E(\rho,\delta)$ as a function of density $\rho = \rho_{\mathrm{n}} + \rho_{\mathrm{p}}$ and isospin asymmetry $\delta = (\rho_{\mathrm{n}} - \rho_{\mathrm{p}})/\rho$ with ρ_{n} (ρ_{p}) being the neutron (proton) density. The parabolic approximation to nuclear matter EOS, i.e., $E(\rho,\delta) \approx E(\rho,\delta=0) + E_{\mathrm{sym}}(\rho)\delta^2$, is adopted widely with the symmetry energy defined as $E_{\mathrm{sym}}(\rho) = \frac{1}{2!}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}|_{\delta=0}$. The feasibility of the parabolic approximation is practically justified in various aspects of nuclear physics, especially in finite nuclei where the δ^2 value is usually significantly less than one. Nevertheless, in neutron stars where the δ could be close to one, a sizable higher-order terms of isospin dependent part of nuclear matter EOS, e.g., the term $E_{\mathrm{sym},4}(\rho)\delta^4$ with the fourth-order symmetry energy defined as $E_{\mathrm{sym},4}(\rho) = \frac{1}{4!}\frac{\partial^4 E(\rho,\delta)}{\partial\delta^4}|_{\delta=0}$, may have substantial effects on the properties such as the proton fraction at beta-equilibrium, the core-crust transition density and the critical density for the direct URCA process [13–17].

To the best of our knowledge, unfortunately, there is so far essentially no experimental information on the magnitude of $E_{\mathrm{sym},4}(\rho)$, even at normal nuclear density ρ_0. Theoretically, the mean-field models generally predict the magnitude of $E_{\mathrm{sym},4}(\rho_0)$ is less than 2 MeV [16,18–20]. A value of $E_{\mathrm{sym},4}(\rho_0) = 1.5$ MeV is obtained from chiral pion–nucleon dynamics [21]. The recent study [22] within the quantum molecular dynamics (QMD) model indicates that the $E_{\mathrm{sym},4}(\rho_0)$ could be as large as $3.27 \sim 12.7$ MeV depending on the interactions used. Based on an interacting Fermi gas model, a significant value of 7.18 ± 2.52 MeV [23] is predicted for the kinetic part of $E_{\mathrm{sym},4}(\rho_0)$ by considering the high-momentum tail [24] in the single-nucleon momentum distributions that could be due to short-range correlations of nucleon–nucleon interactions. In addition, the divergence of the isospin-asymmetry expansion of nuclear matter EOS in many-body perturbation theory is discussed in Refs. [21,25]. Therefore, the magnitude of $E_{\mathrm{sym},4}(\rho_0)$ is currently largely uncertain and it is of critical importance to obtain some experimental or empirical information on $E_{\mathrm{sym},4}(\rho_0)$.

Conventionally nuclear matter EOS is quantitatively characterized in terms of a few characteristic coefficients through Taylor expansion in density at ρ_0, e.g., $E(\rho,\delta=0) = E_0(\rho_0) + \frac{1}{2!}K_0\chi^2 + \frac{1}{3!}J_0\chi^3 + \mathcal{O}(\chi^4)$ and $E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}(\rho_0) + L\chi + \frac{1}{2!}K_{\mathrm{sym}}\chi^2 +$

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$\mathcal{O}(\chi^3)$ with $\chi = \frac{\rho-\rho_0}{3\rho_0}$. The density in the interior of heavy nuclei is believed to nicely approximate to saturation density of symmetric nuclear matter (nuclear normal density) ρ_0 and the empirical value of $\rho_0 \approx 0.16$ fm^{-3} has been obtained from measurements on electron or nucleon scattering off heavy nuclei [26]. Our knowledge on nuclear matter EOS largely stems from nuclear masses based on various nuclear mass formulae. By analyzing the data on nuclear masses with various nuclear mass formulae (see, e.g., Ref. [27]), consensus has been reached on $E_0(\rho_0)$ and $E_{\rm sym}(\rho_0)$ with $E_0(\rho_0) \approx -16.0$ MeV and $E_{\rm sym}(\rho_0) \approx 32.0$ MeV. These empirical values on $E_0(\rho_0)$ and $E_{\rm sym}(\rho_0)$ are of critical importance for our understanding on nuclear matter EOS.

Generally speaking, it is very hard to determine the higher-order parameter $E_{\rm sym,4}(\rho_0)$ and the fourth-order symmetry energy $a_{\rm sym,4}(A)$ of finite nuclei from simply fitting nuclear masses within nuclear mass formulae since the term $a_{\rm sym,4}(A)I^4$ ($I = \frac{N-Z}{A}$ with N and Z being the neutron and proton number, respectively, and $A = N + Z$ is mass number) is considerably small compared to other lower-order terms in the mass formula for known nuclei, even for the predicted dripline nuclei [28]. Recently, however, by approximating $a_{\rm sym,4}(A)$ to a constant $c_{\rm sym,4}$ in the mass formula, several studies [29–32] have been performed to extract $c_{\rm sym,4}$ from analyzing the double difference of the "experimental" symmetry energy extracted from nuclear mass data, and robust results with high precision have been obtained, i.e., a sizable positive value of $c_{\rm sym,4} = 3.28 \pm 0.50$ MeV or 8.47 ± 0.49 MeV is obtained in Ref. [29], depending on the Wigner term form in the mass formula. More recently, a value of $c_{\rm sym,4} = 8.33 \pm 1.21$ MeV is extracted in Ref. [32] using similar analysis on nuclear masses. These results provide the possibility to extract information on $E_{\rm sym,4}(\rho_0)$.

In this work, by self-consistently considering the bulk, surface and Coulomb contributions to the nuclear mass, we extend the mass formula of Ref. [33] to additionally include the corrections due to central density variation of finite nuclei and the higher-order fourth-order symmetry energy term $a_{\rm sym,4}(A)I^4$. In this extended mass formula, a explicit relation between $a_{\rm sym,4}(A)$ and $E_{\rm sym,4}(\rho_0)$ is obtained. We demonstrate for the first time that the precise value of $c_{\rm sym,4}$ obtained recently from nuclear mass analysis allows us to estimate a value of $E_{\rm sym,4}(\rho_0) = 20.0 \pm 4.6$ MeV.

2. Nuclear mass formula

There have been a number of nuclear mass models which aim to describe the experimental nuclear mass database and predict the mass of unknown nuclei. Nowadays, some sophisticated mass formulae [27,35–37] (with shell and pairing corrections) can reproduce the measured masses of more than 2000 nuclei with a root-mean-square deviation of merely several hundred keVs. These mass formulae provide us empirical information about the EOS of nuclear matter, especially its lower-order characteristic parameters $E_0(\rho_0)$, $E_{\rm sym}(\rho_0)$ and so forth.

To relate the coefficients in the mass formula to the EOS of nuclear matter, one can express the binding energy $B(N, Z)$ of a nucleus with N neutrons and Z protons in terms of the bulk energy of nuclear matter in the interior of the nucleus plus surface corrections and Coulomb energy. Based on such an argument, Danielewicz [33] developed a mass formula with a self-consistent A-dependent symmetry energy $a_{\rm sym}(A)$ of finite nuclei. Considering that the central density $\rho_{\rm cen}$ in nuclei generally depends on N and Z and deviates from ρ_0, we here extend the mass formula of Ref. [33] by considering the deviation of $\rho_{\rm cen}$ from ρ_0, and additionally including the higher-order I^4 terms. In such a framework, a nucleus with N neutrons and Z protons is assumed to localize inside an effective sharp radius R, i.e.,

$$R = r_0\left[1 + 3\chi_{\rm cen}(N, Z)\right]^{-1/3} A^{1/3}, \tag{1}$$

where r_0 is a constant satisfying $\frac{4}{3}\pi\rho_0 r_0^3 = 1$ and $\chi_{\rm cen} = (\rho_{\rm cen} - \rho_0)/3\rho_0$ is a dimensionless variable characterizing the deviation of $\rho_{\rm cen}$ from ρ_0. Furthermore, we denote the volume (surface) neutron excess as $\Delta_{\rm v} = N_{\rm v} - Z_{\rm v}$ ($\Delta_{\rm s} = N_{\rm s} - Z_{\rm s}$), where $N_{\rm v}$ ($Z_{\rm v}$) and $N_{\rm s}$ ($Z_{\rm s}$) represent the neutron (proton) number in the volume and surface regions of the nucleus, respectively, with $N_{\rm v} + N_{\rm s} = N$ and $Z_{\rm v} + Z_{\rm s} = Z$. Generally, $\chi_{\rm cen}$ and $\Delta_{\rm v}$ ($\Delta_{\rm s}$) depend on N and Z of the nucleus and can be determined from equilibrium conditions, and this is consistent with the argument of the droplet model (see, e.g., Ref. [34]).

In the present work, the nuclear binding energy consists of volume term $B_{\rm v}$, surface term $B_{\rm s}$ and Coulomb term $B_{\rm c}$. The volume part of the binding energy can be treated in nuclear matter approximation, i.e.,

$$\begin{aligned} B_{\rm v} \approx A\Big[&E_0(\rho_0) + \frac{1}{2}K_0\chi_{\rm cen}^2 + E_{\rm sym}(\rho_0)\Big(\frac{\Delta_{\rm v}}{A}\Big)^2 \\ &+ L\chi_{\rm cen}\Big(\frac{\Delta_{\rm v}}{A}\Big)^2 + E_{\rm sym,4}(\rho_0)\Big(\frac{\Delta_{\rm v}}{A}\Big)^4\Big]. \end{aligned} \tag{2}$$

The surface term comes from surface tension and symmetry potential (detailed argument can be found in Ref. [33]), and it can be expressed as

$$\begin{aligned} B_{\rm s} &= \Big[\sigma_0 - \sigma_{\rm I}\Big(\frac{\Delta_{\rm s}}{S}\Big)^2\Big]4\pi R^2 + \frac{2\sigma_{\rm I}}{4\pi R^2}\Delta_{\rm s}^2 \\ &\approx E_{\rm s0}(1 - 2\chi_{\rm cen})A^{\frac{2}{3}} + \beta(1 + 2\chi_{\rm cen})A^{\frac{4}{3}}\Big(\frac{\Delta_{\rm s}}{A}\Big)^2, \end{aligned} \tag{3}$$

where σ_0 ($\sigma_{\rm I}$) represents the isospin independent (dependent) surface tension, $S = 4\pi R^2$ is the surface area of the nucleus, and we define $E_{\rm s0} = 4\pi r_0^2\sigma_0$ and $\beta = \frac{\sigma_{\rm I}}{4\pi r_0^2}$. Eq. (1) has been used to obtain the second line in Eq. (3). For Coulomb energy, for simplicity we adopt the following simple form without exchange term, i.e.,

$$B_{\rm c} = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\frac{1}{R}Z^2 \approx a_c A^{-1/3} Z^2(1 + \chi_{\rm cen}), \tag{4}$$

with $a_c = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0 r_0}$.

The equilibrium condition of nuclei can be obtained from variations of the binding energy $B(N, Z)$ of the nucleus with respect to $\chi_{\rm cen}$ and $\Delta_{\rm v}$, i.e.,

$$\frac{\partial B(N, Z)}{\partial\chi_{\rm cen}} = 0, \qquad \frac{\partial B(N, Z)}{\partial\Delta_{\rm v}} = 0, \tag{5}$$

from which we can obtain $\chi_{\rm cen}$ and $\Delta_{\rm v}$ ($\Delta_{\rm s}$) for different A and Z. The first equation means the mechanical equilibrium and tells us how the surface energy, Coulomb energy and the isospin dependent part of volume energy affect the value of $\rho_{\rm cen}$, while the second equation represents the balance of the isospin asymmetry chemical potential between the volume and surface regions.

To solve Eq. (5), we expand $\chi_{\rm cen}$ in terms of $\frac{\Delta_{\rm v}}{A}$, and then expand $(\frac{\Delta_{\rm v}}{A})^2$ in terms of I, i.e.,

$$\chi_{\rm cen} = \chi_0 + \chi_2\Big(\frac{\Delta_{\rm v}}{A}\Big)^2 + \mathcal{O}\Big[\Big(\frac{\Delta_{\rm v}}{A}\Big)^4\Big], \tag{6}$$

$$\Big(\frac{\Delta_{\rm v}}{A}\Big)^2 = D_2 I^2 + \mathcal{O}(I^4), \tag{7}$$

where the expansion coefficients χ_0, χ_2 and D_2 might depend on A or Z, consistent with calculations from the droplet model [34] and the Thomas–Fermi approximation [38]. Using Eqs. (2), (3) and