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Renormalized quark–antiquark Hamiltonian induced by a gluon mass ansatz in heavy-flavor QCD

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ABSTRACT

In response to the growing need for theoretical tools that can be used in QCD to describe and understand the dynamics of gluons in hadrons in the Minkowski space–time, the renormalization group procedure for effective particles (RGPEP) is shown in the simplest available context of heavy quarkonia to exhibit a welcome degree of universality in the first approximation it yields once one assumes that beyond perturbation theory gluons obtain effective mass. Namely, in the second-order terms, the Coulomb potential with Breit–Fermi spin couplings in the effective quark–antiquark component of a heavy quarkonium, is corrected in one-flavor QCD by a spin-independent harmonic oscillator term that does not depend on the assumed effective gluon mass or the choice of the RGPEP generator. The new generator we use here is much simpler than the ones used before and has the advantage of being suitable for studies of the effective gluon dynamics at higher orders than the second and beyond the perturbative expansion.

1. Introduction

The growing need for understanding dynamics of gluons in QCD, comprehensively documented in [1], revives interest in Hamiltonian methods for describing heavy quarkonia in terms of wave functions in the Fock space of virtual quanta, where gluons are likely to behave in a relatively simple way because they are permanently coupled to the massive quarks that move slowly with respect to each other. Phenomenology and theory of heavy quarkonia rapidly develops, as is illustrated by many examples [2–12]. The key feature regarding gluons that requires understanding is how they acquire the effective mass, so that the hadron mass spectra [13] do not exhibit any small excitations such as the atomic spectra do due to massless photons. The gluon-mass generation is a subject of research from early on using continuum Dyson–Schwinger equations [14–16] and it is addressed in lattice studies [17–20] because its implications for theory of strong interactions would be broad, including the issue of saturation in dense gluon systems beyond a single hadron setting [21]. In the canonical formulation of QCD in the front form (FF) of dynamics in the Minkowski space–time [22], the need for understanding implications of an effective gluon mass is stressed in general in [23] and in the context of heavy quarkonia in [24,25]. Theoretical studies of heavy quarkonia may thus be expected to increasingly focus on the Hamiltonian dynamics of their gluonic content, *cf.* [26,27]. This article presents a first step in a program of systematic studies of dynamics of scale-dependent gluons in heavy quarkonia, starting with the simplified case of canonical FF formulation of QCD with quarks of just one heavy flavor and assuming that the effective gluons which correspond to momentum scale of quark binding mechanism in quarkonia have mass.

We calculate a renormalized, scale-dependent Hamiltonian for quarkonium constituents in the Fock space using a new formulation of the renormalization group procedure for effective particles (RGPEP) in quantum field theory, see below. In this new formulation, the key RGPEP element called its generator does not depend on the derivative of the renormalized Hamiltonian with respect to the scale parameter, in contrast to the earlier versions of the RGPEP, in which such dependence was involved. The generator dependence on the derivative of the Hamiltonian made a calculation of the latter difficult and stalled the development. With the derivative issue resolved, the improved method is now prepared for trial applications and further development. The new generator has been

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already verified to work beyond perturbative expansion in simple theories [28,29] and it has passed the test of producing asymptotic freedom in the renormalized FF Hamiltonian of QCD [30]. Here we present the new-generator calculation of a renormalized Hamiltonian in one-flavor theory including terms up to second order in powers of the strong-interaction coupling constant at the scale of heavy quarkonia.

The renormalized Hamiltonian at a suitable scale is subsequently applied to formulate the eigenvalue problem for a quarkonium in the Fock space basis that is constructed using the creation operators for effective particles of the corresponding size. We make a drastically simplifying assumption that the non-Abelian and non-perturbative effects due to components with more effective gluons than one, can be mimicked by a gluon-mass ansatz introduced in the dynamics of the component with only one effective gluon and a quark–antiquark pair. Namely, the components with more than one effective gluon are dropped at the price of introducing the ansatz. We then express the quark–antiquark–gluon component in terms of the quark–antiquark component and calculate the effective Hamiltonian in the quark–antiquark sector. As a result, we establish the universality of effective interaction one obtains in the quark–antiquark component: the new RGPEP generator produces the same Coulomb term with Breit–Fermi spin-dependent factors and the same spin-independent harmonic oscillator term that were previously obtained with the old generator [31,32]. The oscillator term is independent of the value of the mass ansatz.

The new effective term respects rotational symmetry in the quark–antiquark relative three-momentum space, in which the eigenvalue problem for quarkonium two-body component resembles a non-relativistic Schrödinger equation with a potential, except that the eigenvalue is the quarkonium mass squared instead of its energy. The effective eigenvalue equation is invariant with respect to the FF kinematical Lorentz transformations. We also note that the relative momentum variables we use for heavy quarkonia resemble the variables used in the light-front holographic approach to the phenomenology of light hadrons, based on the AdS/QCD duality ideas [33]. The holographic potential is also of the harmonic oscillator form, but its frequency is different, which can be associated with much smaller mass of light quarks than Λ_{QCD} while the heavy quark mass is much greater.

Section 2 explains the preliminary nature of the gluon mass ansatz that is used to finesse the effective quark–antiquark interaction. Section 3 presents the RGPEP in application to one-flavor QCD. The renormalized eigenvalue equation for the entire quarkonium state and the effective Hamiltonian for its quark–antiquark component are introduced in Section 4. Section 5 shows how the effective harmonic oscillator potential emerges in the non-relativistic limit, using holographic quark–antiquark relative momentum variables. Section 6 concludes the article with comments on how our results prepare ground for renormalized Hamiltonian studies of gluon dynamics in heavy quarkonia.

2. The initial need for gluon mass ansatz

The difficulty of describing bound states in the Fock space is that one needs to deal with an *a priori* infinite number of components. In the case of quarkonium, the bound state

$$|\psi\rangle = |Q\,\bar{Q}\rangle + |Q\,\bar{Q}\,G\rangle + |Q\,\bar{Q}\,GG\rangle + \dots\,, \tag{1}$$

is built from canonical quanta of quark and gluon fields. To deal with this issue, the RGPEP uses the concept of effective particles. They are characterized by an effective size s and are related to the bare, point-like particles of canonical theory by means of an operator transformation. The idea is that for description of observables characterized by the momentum scale $\lambda = 1/s$, one can use the renormalized Hamiltonian H_s, so that the number of relevant Fock components in Eq. (1) is sufficiently small for carrying out computations, except that Q, $\bar{Q}$ and G are replaced by Q_s, $\bar{Q}_s$ and G_s, respectively. Thus, infinitely many Fock components for effective particles can be neglected. The bound-state problem becomes so greatly simplified that one can attempt to seek solutions to the eigenvalue equation numerically.

The above strategy may work when the RGPEP equation for H_s is solved exactly. When one uses expansions of H_s in a series of powers of effective coupling constant, non-perturbative effects in H_s itself are not included. The eigenvalue problem for such H_s still couples the Fock components made of effective particles in significant ways. Initially, one cannot be certain that the Fock components with more than one effective gluon can be dropped from the eigenvalue problem with no consequence. Since power counting allows a mass term for gluons, perturbative RGPEP calculations imply a need for a gluon mass counterterm in the canonical Hamiltonian of one-flavor QCD, whose finite part is not known, and the phenomenology of hadrons appears to exclude massless effective gluons, one is in need to assume that neglecting components with more than one effective gluon of size s in heavy quarkonia of characteristic physical size s may only be admissible if one allows for appearance of the effective mass term for the gluon in the $Q_s\bar{Q}_sG_s$ component, presumably produced by the non-Abelian interaction effects descendant from higher components, absent in Abelian theories.

In the variety of terms that are conceivable in renormalized Hamiltonians using FF power counting [23], rotational symmetry is not explicitly respected. Any ansatz one introduces to hypothetically account for the omitted terms, must satisfy the condition that the resulting effective eigenvalue problem produces the mass spectrum that exhibits degeneracy implied by rotational symmetry. Making an ansatz for the gluon mass term, rather than any other potentially allowed term, turns out to lead to the effective oscillator correction to the Coulomb terms that satisfies the rotational-symmetry condition already at the level of effective Hamiltonian itself, exhibiting the symmetry in relative momentum variables. It is unlikely that ansatz terms for quantities other than mass can easily produce such a universal result. Moreover, the mass ansatz turns out to possess the unique feature that the oscillator potential it leads to is independent of the actual value of the ansatz mass, provided it is not too small.

When the RGPEP calculation of renormalized Hamiltonian H_s is extended to higher orders than second, and when the unknowns of the effective gluon mass are relegated to the Fock sectors with more than one gluon, the preliminary gluon-mass ansatz in the dynamics of $Q_s\bar{Q}_sG_s$ component will be replaced by true QCD terms. However, since the complexity of renormalized Hamiltonian operators requires approximations, and since their spectra are calculated numerically making further simplifications, a purely mathematical approach to identification of all important terms may turn out hard to execute. Fortunately, the spectra of Hamiltonians for heavy quarkonia in the Fock space representation with effective gluons can also be studied in comparison with experimental data. Assuming that the canonical theory one starts from is right, one can use the data in identifying scale dependence of various terms allowed by the power counting. The mass ansatz seems to be the simplest admissible term to falsify. The universality of the quarkonium effective Hamiltonian described here prepares the ground for required theoretical studies in parallel with quarkonium phenomenology.