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Proper temperature of the Schwarzschild AdS black hole revisited

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ABSTRACT

The Unruh temperature calculated by using the global embedding of the Schwarzschild AdS spacetime into the Minkowski spacetime was identified with the local proper temperature; however, it became imaginary in a certain region outside the event horizon. So, the temperature was assumed to be zero of non-thermal radiation for that region. In this work, we revisit this issue in an exactly soluble two-dimensional Schwarzschild AdS black hole and present an alternative resolution to this problem in terms of the Tolman's procedure. However, the process appears to be non-trivial in the sense that the original procedure assuming the traceless energy–momentum tensor should be extended in such a way that it should cover the non-vanishing case of the energy–momentum tensor in the presence of the trace anomaly. Consequently, we show that the proper temperature turns out to be real everywhere outside the event horizon without any imaginary value, in particular, it vanishes at both the horizon and the asymptotic infinity.

1. Introduction

One of the most outstanding theoretical results in quantum mechanics of black holes would be Hawking radiation [1,2]. On general grounds, the thermal distribution of Hawking radiation could be characterized by the two black hole temperatures associated with the observers. One is the fiducial temperature measured by a fixed observer who undergoes acceleration, which is usually given as the redshifted Hawking temperature at a finite distance outside the event horizon [3]. The other is the proper temperature measured by a freely falling observer from rest, which is expressed by the Tolman temperature [4,5].

Surprisingly, the fiducial temperature takes the same form as the proper temperature even though the respective observers belong to different frames. At first glance, one might conclude that the equivalence principle could be violated, particularly at the horizon from the fact that the Tolman temperature could be divergent there. However, it is worth noting that the equivalence principle could be restored just at the horizon as seen from the calculations using the particle detector method [6]. This fact could also be confirmed by showing that the Tolman temperature could vanish effectively at the horizon [7].

As a matter of fact, the Tolman temperature for the freely falling observer should be modified effectively, so that its behavior could be shown to be definitely different from that of the fiducial temperature [7]. Recently, a similar argument for the proper temperature [8] could also be obtained from a different point of view by clarifying the Hawking effect [1] and the Unruh effect [9]. Note that all these arguments are for asymptotically flat black holes, and it appears to be natural to ask how to get the proper temperatures in asymptotically non-flat spacetimes such as the Schwarzschild anti-de Sitter (SAdS) black hole.

Regarding the calculations of the proper temperature in the SAdS black hole, there has been pioneering works employing the global embedding in Minkowski spacetime (GEMS) approach, where an accelerating observer in a higher-dimensional Minkowski spacetime perceives thermal radiation characterized by the Unruh temperature which will be identified with the proper temperature in the original spacetime [10,11]; however, the proper temperature suffers from an imaginary value. So, it was claimed that the imaginary valued proper temperature would indicate non-thermal radiation [12]. The non-thermal condition to evade the imaginary temperature seems to be somehow *ad hoc*. Obviously, the temperature could be made real in the near horizon limit when a reduced embedding was used [13].

Now it raises a question: is there any other way to resolve this imaginary value problem for the proper temperature in the SAdS

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black hole? In fact, there is another way to calculate the proper temperature directly, which is the old-fashioned but clear Tolman procedure [4,5], which might provide a plausible solution to this question. However, this approach appears to be conceptually nontrivial in the sense that the conventional Tolman temperature derived from the conventional Stefan–Boltzmann law rests upon the traceless condition of the energy–momentum tensor. If one were to study the proper temperature on the background of the asymptotically anti-de Sitter (AdS) spacetimes, the traceless condition for the energy–momentum tensor should be released in order to take into account the non-vanishing trace of the quantized energy–momentum tensor in the presence of the trace anomaly.

In this work, we would like to revisit the proper temperature of the SAdS black hole and show how to get the well-defined real-valued proper temperature. In essence, we shall obtain a modified Stefan–Boltzmann law, which is commensurate with the presence of the non-vanishing trace of the energy–momentum tensor. Then, from the modified Stefan–Boltzmann law, we shall derive an effective Tolman temperature and obtain the desired result. In fact, such a modification of the Stefan–Boltzmann law has already been applied to various models: thermodynamics of particle physics in flat spacetime [14], thermodynamics of black hole in curved spacetime [7], and warm inflation models in cosmology [15], so that some puzzling problems have been successfully resolved.

Our calculations will be done in a two-dimensional amenable model in order to solve exactly without losing any essential physics. In Sec. 2, the proper temperature will be obtained in the two-dimensional SAdS black hole by using the GEMS [10–12] in the self-contained manner in comparison with our result. As expected, we find that the imaginary temperature is unavoidable in a certain region. Next, in Sec. 3, we will calculate the proper temperature from the Tolman's procedure [4,5] by releasing the traceless condition for the energy–momentum tensor. We shall show that the proper temperature for the SAdS black hole turns out to be real everywhere outside the horizon without any imaginary value, so that it becomes smooth without any cusp. Summary and discussion will be given in Sec. 4.

2. Proper temperature from the GEMS

We recapitulate how the proper temperature for the two-dimensional SAdS black hole could be derived from the framework of the GEMS employed in Ref. [12]. Let us start with the two-dimensional SAdS black hole described by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \tag{1}$$

where $f(r) = 1 - 2M/r + r^2/\ell^2$. The metric element can be rewritten as

$$f(r) = \frac{1}{\ell^2}\left(1 - \frac{r_h}{r}\right)(r^2 + rr_h + r_h^2 + \ell^2), \tag{2}$$

where r_h is the horizon of the black hole and the mass is related to the horizon as

$$M = \frac{r_h}{2\ell^2}(r_h^2 + \ell^2). \tag{3}$$

And the surface gravity is also given by

$$\kappa = \frac{f'(r_h)}{2} = \frac{3r_h^2 + \ell^2}{2\ell^2 r_h}, \tag{4}$$

where the prime denotes the derivative with respect to r.

Performing the global embedding of the SAdS spacetime into the higher dimensional Minkowski spacetime, a free-fall observer on the SAdS black hole could be identified with the accelerated observer in the higher dimensional Rindler spacetime [10,12], so that the Rindler observer could find the Unruh temperature as [9]

$$T = \frac{a}{2\pi}, \tag{5}$$

where a is the proper acceleration of the observer in the higher dimensional Minkowski spacetime.

The higher dimensional Minkowski spacetime can be obtained by the following transformation [11]

$$\begin{aligned}
X^0 &= \kappa^{-1}\sqrt{f}\sinh\kappa t, \qquad X^1 = \kappa^{-1}\sqrt{f}\cosh\kappa t, \qquad X^2 = r,\\
X^3 &= \int dr \frac{\ell(r_h^2+\ell^2)}{3r_h^2+\ell^2}\sqrt{\frac{r_h(r^2+rr_h+r_h^2)}{r^3(r^2+rr_h+r_h^2+\ell^2)}},\\
X^4 &= \int dr \frac{1}{3r_h^2+\ell^2}\sqrt{\frac{(9r_h^4+10r_h^2\ell^2+\ell^4)(r^2+rr_h+r_h^2)}{r^2+rr_h+r_h^2+\ell^2}},
\end{aligned} \tag{6}$$

where the line element is $ds^2 = \eta_{IJ}dX^I dX^J$ with $\eta_{IJ} = \mathrm{diag}(-1, 1, 1, 1, -1)$. In this spacetime, the square of the proper acceleration is calculated as

$$\begin{aligned}
a^2 &= \eta_{IJ}a^I a^J\\
&= \frac{[2+(3+c^2)x+(1+c^2)x^3][-2+(1+c^2)(1+x)x]}{4\ell^2[1+x+(1+c^2)x^2]},
\end{aligned} \tag{7}$$

where $x = r_h/r$ and $c = \ell/r_h$. Substituting this acceleration (7) into Eq. (5), the Unruh temperature regarded as the proper temperature is obtained as

$$T = \frac{1}{4\pi\ell}\frac{\sqrt{[2+(3+c^2)x+(1+c^2)x^3][-2+(1+c^2)(1+x)x]}}{\sqrt{1+x+(1+c^2)x^2}}. \tag{8}$$

The squared proper temperature is positive $r < r_c$, while it is negative for $r > r_c$ where the critical radius is given by $r_c = (r_h^2 + \ell^2 + \sqrt{9r_h^4 + 10r_h^2\ell^2 + \ell^4})/(4r_h)$. In particular, the squared temperature at the horizon becomes

$$T^2(r_h) = \frac{1}{4\pi^2}\left[\frac{\ell^2}{2r_h^4} + \frac{3}{3r_h^2+\ell^2}\right], \tag{9}$$

which is positive finite. By the way, at the asymptotic infinity, the squared temperature takes the form of

$$T^2(\infty) \to -\frac{1}{4\pi^2}\left[\frac{1}{\ell^2} + \frac{(r_h^2+\ell^2)^2}{2r_h^4\ell^2}\right], \tag{10}$$

which is negative. Thus one can find that the proper temperature becomes imaginary for $r > r_c$. In fact, it was claimed that the imaginary proper temperature would indicate non-thermal radiation [12], and the proper temperature was assumed to be zero for $r > r_c$. It means that there would appear a cusp at r_c for the temperature curve. In the next section, we will find another way to resolve this imaginary value problem by directly calculating the proper temperature through the Tolman's procedure.

3. Proper temperature from the Tolman procedure

We calculate the temperature measured by a freely falling observer released from rest on the SAdS black hole. Here, we shall release the traceless condition employed in the conventional formulation of the Tolman temperature [4,5] in order to get the effective Tolman temperature.