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Far-from-equilibrium initial conditions probed by a nonlocal observable

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ABSTRACT

Using the gauge/gravity duality, we investigate the evolution of an out-of-equilibrium strongly-coupled plasma from the viewpoint of the two-point function of scalar gauge-invariant operators with large conformal dimension. This system is out of equilibrium due to the presence of anisotropy and/or a massive scalar field. Considering various functions for the initial anisotropy and scalar field, we conclude that the effect of the anisotropy on the evolution of the two-point function is considerably more than the effect of the scalar field. We also show that the ordering of the equilibration time of the one-point function for the non-probe scalar field and the correlation function between two points with a fixed separation can be reversed by changing the initial configuration of the plasma, when the system is out of the equilibrium due to the presence of at least two different sources like our problem. In addition, we find the equilibration time of the two-point function to be linearly increasing with respect to the separation of the two points with a fixed slope, regardless of the initial configuration that we start with. Finally we observe that, for larger separations the geodesic connecting two points on the boundary crosses the event horizon after it has reached its final equilibrium value, meaning that the two-point function can probe behind the event horizon.

1. Introduction

Understanding the out-of-equilibrium dynamics of a strongly-coupled gauge theory has attracted much interest in the past decade. One motivation for this comes from the results obtained in the ultra-relativistic heavy ion collisions at the Brookhaven RHIC and CERN LHC. The experimental observations, such as the elliptic flow, indicate that the hot and dense medium, created at the early moments of the collisions, is more similar to a perfect fluid with a small entropy-normalized viscosity than a collection of the individual partons [1,2]. This medium called Quark-Gluon Plasma (QGP) is a strongly-coupled phase of QCD which is highly out of equilibrium and takes a very short time of order 1 fm/c to reach a thermalized state [3]. A challenging and still unresolved question is how a strongly-coupled out-of-equilibrium gauge theory evolves towards its thermal equilibrium.

A powerful tool to study the dynamics of such systems is the gauge/gravity duality [4–7] which solves the previously intractable problems using the classical gravitational dynamics of the asymptotically anti-de Sitter spacetimes. Plenty of work has been done on the early-time dynamics and subsequent thermalization in the past decade using various holographic techniques [8–24]. In particular, one way for investigating the dynamics of strongly-coupled far-from-equilibrium systems proposed in [10,11,19] is to drive the field theory out of equilibrium by distorting the metric for a finite amount of time in such a way that an anisotropy is created between the longitudinal and transverse directions. By solving the Einstein equations using the elegant method of characteristic formulation, they were able to study the isotropization of the system. Another way for investigating the early-time dynamics of far-from-equilibrium systems is to start from an out-of-equilibrium plasma, in the absence of external sources, and then allow it to evolve and eventually reach its thermal equilibrium [14,16]. In this approach the boundary metric is flat and time-independent, and the energy

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density is conserved. If the initial state is anisotropic, the pressure in the longitudinal and transverse directions will evolve in time and hence we are able to study the isotropization process.

One can also add an operator to the field theory and observe the time evolution of its one-point function for studying the thermalization process. In [22] we followed the second approach considering an initially anisotropic geometry interacting with a non-probe massive scalar field. We supposed that the expectation value of the non-probe scalar field at equilibrium is zero. Computing the pressure anisotropy and expectation value of the non-probe scalar field as time proceeds, we were able to study the process of approaching the equilibrium. For our purposes in most of the calculations we employed a Gaussian profile with tractable parameters as the initial function for both the non-probe scalar field and anisotropy function. By changing the parameters, we concluded that depending on the configuration of the initial plasma, various ordering of relaxation times, i.e. isotropization, thermalization and equilibration times, are possible. As introduced in [22], by these three time-scales we mean the time at which longitudinal and transverse pressures become equal, the entropy production ends, and the expectation value of the non-probe scalar field reaches its equilibrium, respectively, within the numerical precision chosen.

Extended objects such as correlation function of two local gauge-invariant operators, expectation values of Wilson loops and entanglement entropy are more sensible to the details of the thermalization process than local probes [25–32]. Providing an elegant geometric way to compute these observables, AdS/CFT gives an explanation for this statement; nonlocal probes penetrate deeper into the bulk, hence probe a wider range of energies in the boundary field theory. In fact, since nonlocal probes distinguish different scales, they can be used for considering the contribution of different scales of energy in the thermalization process.

Our purpose in the present paper is to investigate the thermalization process by studying the time evolution of both local and nonlocal observables. To that purpose, we consider the solution obtained in [22] as an out-of-equilibrium background. On the gravity side, this system is an initially anisotropic geometry interacting with a non-probe scalar field dual to a scalar operator of conformal dimension $\Delta = 3$. Using numerical calculation we find the time-evolution of the expectation value of this scalar operator. Moreover, we calculate the equal-time two-point correlation function for another scalar operator which has a large conformal dimension $\Delta \gg 1$ and its backreaction to the field theory is not taken into account. According to the AdS/CFT dictionary, to obtain the evolution of the two-point function in the field theory, we compute the time evolution of the geodesic length connecting the two points where the local operators are inserted at the boundary. Changing the initial configuration of the system, we would be able to study the evolution of the two-point function for different time ordering of the relaxation time-scales. The main difference between our problem with the problems in [30,32] is the presence of the non-probe scalar field beside the anisotropy. This enables us to study the effect of being out of equilibrium due to more than one source.

The main results we obtain in this paper can be summarized as follows.

- We consider the evolution of several initial configurations, one of them is anisotropic and coupled to a non-probe scalar field and the other ones are isotropic with non-probe scalar field and anisotropic without any scalar field. They are chosen to have the same functions with the same parameters so that they thermalize at considerably similar times, i.e. the area of the event horizon for them reach their final values at similar times. Comparing the evolution of their two-point functions, we found the effect of the scalar operator on the two-point function to be much less than the anisotropy. The non-probe scalar field has considerable effects on the two-point function evolution, only when it is chosen to be very far from equilibrium, i.e. its initial function concentrate near the event horizon.
- We show that the ordering of the equilibration of the non-probe scalar one-point function and the probe two-point function can be reversed, by changing the parameters of the initial configuration and this is possible only when there are more than one source for deriving the system out of equilibrium. We should mention here that in [22] we concluded that the ordering of the isotropization, thermalization and equilibration times can be changed by changing the initial configuration. Notice that in the case where there is only one cause for being out of equilibrium, no matter how we change the initial configuration of the system, the ordering of the time-scales remains unchanged. In general, for an out-of-equilibrium system, time ordering of the equilibration of different probes depends on the initial configuration.
- We also find that the equilibration time of the two-point function increases linearly with the separation of the points, for large enough separations, and the slope of this growth is independent of the initial configuration of the system.
- Finally, we show that after the event horizon approaches its equilibrium, the geodesics for large separations on the boundary crosses the event horizon.

The paper is organised as follows. In the next section, we briefly review the calculation of the evolving background metric. Interested readers are referred to [22] and the above-mentioned original papers of the characteristic formulation, for more details. We explain the strategy we use for the calculation of the geodesic length in section 3. Finally, in section 4 we present the results of the numerical calculations for the geodesic length in some graphs and discuss about the thermalization time of the two-point function in different situations.

2. Evolution of an anisotropic spacetime with an scalar field

This section is devoted to a short review of the problem solved in [22]. The metric ansatz for an anisotropic asymptotically AdS_5 spacetime in the generalized Eddington–Finkelstein coordinates can be written in the following form

$$ds_5^2 = 2drdv - A(v,r)dv^2 + \Sigma(v,r)^2\left(e^{-2B(v,r)}dx_L^2 + e^{B(v,r)}d\mathbf{x}_T^2\right). \quad (1)$$

This metic is translationally invariant in the spatial coordinates of the boundary which is located at $r \to \infty$. v is the time coordinate in the bulk and is the boundary time t as $r \to \infty$. The function $B(v,r)$ introduces an anisotropy between the longitudinal (x_L) and transverse ($\mathbf{x}_T$) directions of the boundary. We study the Einstein's general relativity minimally coupled to a massive scalar field with $m^2 = -3$ which is dual to a fermionic mass operator of conformal dimension $\Delta = 3$. By varying the action of the gravity side with respect to the metric and the scalar field and then inserting the ansatz (1) into the resulting equations, we obtain the following equations in terms of the unknown functions $A(v,r)$, $B(v,r)$, $\Sigma(v,r)$ and $\phi(v,r)$,

$$0 = \Sigma\partial_r(\dot{\Sigma}) + 2\dot{\Sigma}\partial_r\Sigma - 2\Sigma^2 + \frac{1}{12}m^2\phi^2\Sigma^2, \quad (2a)$$

$$0 = 2\partial_r(\dot{\phi}) + \frac{3\partial_r\Sigma}{\Sigma}\dot{\phi} + \frac{3\partial_r\phi}{\Sigma}\dot{\Sigma} - m^2\phi, \quad (2b)$$