Physics Letters B 773 (2017) 295–299

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Could the primordial radiation be responsible for vanishing of topological defects?

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ARTICLE INFO

Article history:
Received 1 July 2017
Received in revised form 16 August 2017
Accepted 19 August 2017
Available online 24 August 2017
Editor: M. Trodden

Keywords:
Domain walls
Negative radiation pressure
Solitons

ABSTRACT

We study the motion of domain walls in 1+1, 2+1 and briefly 3+1 d relativistic ϕ^6 model with three equal vacua in the presence of radiation. We show that even small fluctuations can trigger a chain reaction leading to vanishing of the domain walls. Only one vacuum remains stable and domains containing other vacua vanish. We explain this phenomenon in terms of radiation pressure (both positive and negative). We construct an effective model which translates the fluctuations into additional term in the field theory potential. In case of two dimensional model we find a relation between the critical size of the bulk and amplitude of the perturbation.

1. Introduction

Topological defects arise in surprisingly many branches of physics. They can be found in liquid crystals [1], liquid helium [2], ferromagnets, superconductors, graphene [3] and many more important physical substances. It is also natural to expect that topological defects should have been created in large numbers in the early Universe via Kibble–Zurek mechanism during some symmetry breaking phase transitions [4,5]. Unfortunately there are no direct observation evidence proving such objects ever existed. However, it might be plausible that some linear defects (cosmic strings) could give origins to some large scale structures in the Universe. Some observed fluctuations in the cosmic microwave background referred to as "cold spot" could be explained as remnants of textures from early stages the Universe [6,7]. Topological defects are sometimes also considered as one of the dark matter candidates [8]. Surprisingly, there are no signs of other defects like monopoles and domain walls.

Topological defects can interact with each other as well as with some other objects like oscillons [9,10]. A very interesting interaction also can be observed between topological defects and radiation. In some cases the radiation can exert an ordinary radiation pressure proportional to the square of amplitude of incident wave. However, some defects, like kinks in a very popular ϕ^4 theory, are transparent to the radiation in the first order [11,12]. Sine-Gordon kinks are exactly transparent in all orders. Higher order analysis in ϕ^4 model revealed a surprising feature. The kinks undergo the negative radiation pressure (NRP) which accelerates the kinks towards the source of radiation. The acceleration of a kink in ϕ^4 model is proportional to the fourth power of amplitude of the wave. In models with two scalar fields with different masses it is possible that the radiation can exert both positive and negative radiation pressure depending on the composition of the wave [13,14]. In such a case the force exerted on the kink is proportional to the square of the amplitude. More recently some other examples were discussed in case of light and matter waves which, when scattered on a small objects, can bend in such a way that the object would feel the pulling force [15,16]. Mixing between different frequencies can cause the NRP in case of solitons with rotating phase as in Coupled Nonlinear Schrödinger Equation [17]. We want to emphasize that the NRP seen in case of solitons in the present and our previous papers is of a very different nature then the one described in optical physics. It can be exerted on flat and infinite surfaces where simple bending of the light or other wave trajectories is not an option.

In the present paper we consider a mechanism which could increase the rate of the domain wall collisions. In models with at least two equal minima of the potential but with different masses of small perturbations around those vacua. The interest in such models has increased recently [18–20] but they were considered many years ago as for example so called bag models of hadrons

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http://dx.doi.org/10.1016/j.physletb.2017.08.045

[21]. Static kinks (or domain walls in general) in such models have asymmetric profiles. We show that despite the fact that the vacua have the same energies (they are true vacua) the kinks usually accelerate in one direction no matter from where the radiation comes. Antikinks accelerate in opposite direction. Any small perturbation can therefore trigger a chain reaction during which defects collide and create more radiation accelerating other defects causing more collisions.

The present letter is organized as follows. First we define our example ϕ^6 model, than we show how kinks interact with monochromatic wave in case of two vacua with different masses of scalar field. We derive an analytic formula for the force with which such monochromatic wave acts on a kink. Next we show how generic perturbation can influence the stability of kink system (a lattice). In particular we study the effect of random fluctuations with Gaussian distribution filling the whole space. We show that the fluctuations are in some ways equivalent to the shift of the vacua. We also compare the results with other models. The last section concerns higher dimensional case. We find a critical size of a circular domain wall which could either grow or collapse depending on what type of vacuum is inside and how large the fluctuations are.

2. The model

In the present paper we consider one and two dimensional ϕ^6 theory, which can be defined by the rescaled Lagrangian density [22,18]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi), \quad \text{where} \quad U(\phi) = \frac{1}{2}\phi^2\left(\phi^2-1\right)^2. \tag{1}$$

The model has three vacua $\phi_v \in \{-1, 0, 1\}$. Small perturbations around these vacua have different masses: $m_0 = 1$ and $m_1 = 2$ for $\phi_v = 0$ and $\phi_v = \pm 1$ respectively. In one dimensional case the kinks and antikinks can be found from a single solution

$$\phi_K(x) \equiv \phi_{(0,1)}(x) = \sqrt{\frac{1+\tanh x}{2}} \tag{2}$$

using the discrete symmetries of the model: The masses of all kinks are $M = 1/4$. Small perturbation added to the kink solution $\phi(x,t) = \phi_s(x) + \eta(x)e^{i\omega t}$ is governed by a linearized equation

$$-\eta_{xx} + V(x)\eta = \omega^2\eta \tag{3}$$

the potential $V(x)$ is

$$V(x) = U''(\phi_s) = 15\phi_s^4 - 12\phi_s^2 + 1. \tag{4}$$

Note that when $x \to -\infty$ the potential $V \to 1$ and $V(x \to \infty) \to 4$. Solutions to this linearized equation can be found in analytic form in [22]. Let us consider a wave traveling from the left side of kink *i.e.* from $\phi = 0$ vacuum. Asymptotic form of these solutions for frequencies above the two mass thresholds due to Lohe can be written as

$$\begin{cases} \eta_{+\infty}(x) \to e^{ikx}/A(q,k), \\ \eta_{-\infty}(x) \to e^{iqx} + \frac{A(-q,k)}{A(q,k)}e^{-iqx} \end{cases} \tag{5}$$

with

$$q = \sqrt{\omega^2-1},\ k = \sqrt{\omega^2-4},$$
$$A(q,k) = \frac{\Gamma(1-ik)\Gamma(-iq)}{\Gamma(-\frac{1}{2}ik - \frac{1}{2}iq + \frac{5}{2})\Gamma(-\frac{1}{2}ik - \frac{1}{2}iq - \frac{3}{2})}. \tag{6}$$

This solution represents a wave traveling from $-\infty$ with amplitude 1. Amplitude of the reflected wave is equal to $\frac{A(-q,k)}{A(q,k)}$, and amplitude of the transient wave is $\frac{1}{A(q,k)}$. We can use this form to calculate the momentum and energy balance far away from the kink. From Noether's theorem, the conservation laws for energy and momentum density can be written as:

$$\partial_t\mathcal{E} = \partial_x\left(\phi'\dot\phi\right), \tag{7a}$$

$$\partial_t\mathcal{P} = -\frac{1}{2}\partial_x\left(\dot\phi^2 + \phi'^2 - 2U(\phi)\right). \tag{7b}$$

Integrating the above expressions inside interval $[-L, L]$ and averaging over a period T we obtain energy and momentum balance just using asymptotic form of scattering solutions. If the kink does not move initially the conservation laws give (for $\phi = \phi_s + \mathcal{A}\,\mathrm{re}(e^{-i\omega t}\eta(x))$):

$$F_{+\infty}(q,k) = \frac{1}{2}\frac{\mathcal{A}^2}{|A(q,k)|^2}\left(2|A(-q,k)|^2q^2 + qk - k^2\right). \tag{8}$$

We can perform a very similar calculation for the second case when the wave is coming from $+\infty$. The force in this case can be expressed as:

$$F_{-\infty}(q,k) = -F_{+\infty}(k,q) \equiv \mathcal{A}^2(\omega)f(\omega). \tag{9}$$

Fig. 1 shows the force in both cases. Note that the force is positive for all frequencies. The kink will always accelerate towards $+\infty$ no matter which direction the wave comes from. In the first case the wave comes from $\phi = 0$ ($m = 1$) and exerts positive radiation pressure. In the second case it comes from the second vacuum $\phi = 1$ with mass $m = 2$ and exerts negative radiation pressure.

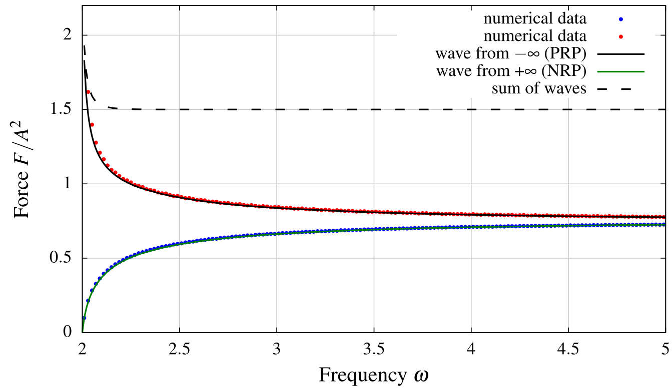

Fig. 1. Theoretical values of the force exerted on the kink. In both cases the force is positive. The color points are the results of numerical calculations for $\mathcal{A} = 0.05$.

3. General perturbations

Kinks interact very weakly with each other on large distances. Their profile vanish exponentially, and so do the interaction between them. For a pair of kinks initially separated by the distance L the estimated time to the collision is of order of $T \approx 2e^{L/2}/\sqrt{L}$. The value was numerically verified. System of static, separated kinks can last even longer, because the forces from neighboring kinks cancel each other (Fig. 2.a, here for the first three kinks $L = 20$ so the timescale to collision is about 10^4). However, adding a small perturbation causes the radiation which exerts pressure on those kinks. We have tested this idea by adding a localized Gaussian profile $\phi = \phi_{kinks} + ae^{-bx^2}$ or colliding two kinks (Fig. 2.b). The collapse of the system was evidently faster compared to the case when no perturbation was added. Because of the polarity in the direction of the radiation pressure, the kinks always accelerate in such a way that the regions with vacuum $\phi = 0$ grow and vacua $\phi = \pm 1$ shrink. Moreover, when kink and antikink collide