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$(1+1)$-dimensional gauge symmetric gravity model and related exact black hole and cosmological solutions in string theory

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ABSTRACT

We introduce a four-dimensional extension of the Poincaré algebra ($\mathcal{N}$) in $(1+1)$-dimensional space-time and obtain a $(1+1)$-dimensional gauge symmetric gravity model using the algebra $\mathcal{N}$. We show that the obtained gravity model is dual (canonically transformed) to the $(1+1)$-dimensional anti de Sitter (*AdS*) gravity. We also obtain some black hole and Friedmann–Robertson–Walker (FRW) solutions by solving its classical equations of motion. Then, we study $\frac{A_{4,8}}{A_1\otimes A_1}$ gauged Wess–Zumino–Witten (WZW) model and obtain some exact black hole and cosmological solutions in string theory. We show that some obtained black hole and cosmological metrics in string theory are same as the metrics obtained in solutions of our gauge symmetric gravity model.

1. Introduction

$(1+1)$-dimensional gravity has been extensively studied by proposing various models. Two of the gravitational theories of most interest are singled out by their simplicity and group theoretical properties. One of them is proposed by Jackiw [1] and Teitelboim [2] (Liouville gravity) which is equivalent to the gauge theory of gravity with (anti) de Sitter group [3–5]. The other one is the string-inspired gravity [6–8] which is equivalent to the gauge theory of the Poincaré group $ISO(1,1)$ [7] and its central extension [9–13].

Recently, two algebras namely the Maxwell algebra [14,15] and the semi-simple extension of the Poincaré algebra [16] have been applied to construct some gauge invariant theories of gravity in four [16–20] and three [21–23] dimensional space-times. These algebras have been also applied to string theory as an internal symmetry of the matter gauge fields [24]. The Maxwell algebra in $(1+1)$-dimensional space-time, is the well-known central extension of the Poincaré algebra which, as we discussed above, has been applied to construct a $(1+1)$-dimensional gauge symmetric gravity action [9,10]. In this paper, we introduce a new four-dimensional extension of the Poincaré algebra ($\mathcal{N}$) in $(1+1)$-dimensional space-time, which is obtained from the 16-dimensional semi-simple extension of Poincaré algebra in $(3+1)$-dimensional space-time [16], by reduction of the dimensions of the space. Then, we construct a $(1+1)$-dimensional gauge symmetric gravity model, using this algebra. We obtain some black hole and cosmological solutions by solving its equations of motion.

On the other hand, in string theory, two-dimensional exact black hole has been found by Witten [6]. Another black hole solution to the string theory has been presented in [25] both in Schwarzschild-like and target space conformal gauges. Exact three-dimensional black string and black hole solutions in string theory have also been found in [26,27]. Here, we study the string theory in $(1+1)$-dimensional space-time, and show that some obtained black hole and cosmological solutions of the gravity model, are exact solutions of the beta function equations (in all loops).

The outline of this paper is as follows: In section 2, we construct a $(1+1)$-dimensional gauge symmetric gravity model using a four-dimensional gauge group related to the algebra $\mathcal{N}$. Then, by presenting a canonical map, we show that the obtained gravity model is dual (canonically transformed) to the $(1+1)$-dimensional *AdS* gravity model. In section 3, we solve the equations of motion and obtain some black hole and Friedmann–Robertson–Walker (FRW) cosmological solutions. Finally, in section 4, we study $\frac{A_{4,8}}{A_1\otimes A_1}$ gauged Wess–Zumino–Witten (WZW) model, and show that some of the resulting string backgrounds, which are exact $(1+1)$-dimensional solutions of the string theory, are the same as the black hole and cosmological solutions obtained for our gravity model. Section five, contains some concluding remarks.

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2. $(1+1)$-dimensional gravity from a non-semi-simple extension of the Poincaré gauge symmetric model

The Poincaré algebra $Iso(1,1)$ in $(1+1)$-dimensional space-time has the following form:

$$[J, P_a] = \epsilon_{ab} P^b, \qquad [P_a, P_b] = 0, \tag{1}$$

where $\epsilon_{01} = -\epsilon^{01} = +1$, and J and P_a $(a = 0,1)$ are generators of the rotation and translations in space-time, and the algebra indices $a = 0,1$ can be raised and lowered by the $(1+1)$-dimensional Minkowski metric η_{ab} $(\eta_{00} = -1,\ \eta_{11} = 1)$ such that $P_a = \eta_{ab} P^b$. In $D = 1+1$, a four-dimensional non-semi-simple extension of the Poincaré algebra[1] $\mathcal{N} = (P_a, J, Z)$ has the following form:

$$[J, P_a] = \epsilon_{ab} P^b, \quad [P_a, P_b] = k\epsilon_{ab} Z, \quad [Z, P_a] = -\frac{\Lambda}{k}\epsilon_{ab} P^b, \tag{2}$$

where Z is the new generator and k and Λ are constants.[2] For $\Lambda = 0$, which leads to $[Z, P_a] = 0$, the above algebra reduces to a solvable algebra which is called the centrally extended Poincaré algebra (or Maxwell algebra[3] in $1+1$ dimensions) [9–11]. We construct the $\mathcal{N}$-algebra valued one-form gauge field as follows:

$$h_i = h_i{}^B X_B = e_i{}^a P_a + \omega_i\, J + A_i\, Z, \qquad i, j = 0, 1 \tag{3}$$

where the indices $i, j = 0,1$ are the space-time indices, and the one-form fields have the following forms:

$$e^a = e_i{}^a dx^i \ , \ \ \omega = \omega_i dx^i \ , \ \ A = A_i dx^i,$$

where $e_i{}^a$, ω_i, A_i are the vierbein, spin connection and the new gauge field, respectively. Using the following infinitesimal gauge parameter:

$$u = \rho^a P_a + \tau\ J + \lambda\ Z,$$

and the gauge transformation as follows:

$$h_i \to h'_i = U^{-1} h_i U + U^{-1}\partial_i U,$$

with $U = e^{-u} \ \simeq\ 1 - u$ and $U^{-1} = e^{u} \ \simeq\ 1 + u$, we obtain the following transformations of the gauge fields:

$$\begin{aligned}
\delta e_i{}^a &= -\partial_i \rho^a - \epsilon^{ab} e_{ib}\,(\tau - \frac{\Lambda}{k}\lambda) + \epsilon^{ab}\rho_b\,(\omega_i - \frac{\Lambda}{k}A_i)\ , \\
\delta\omega_i &= -\partial_i \tau, \\
\delta A_i &= -\partial_i\lambda - k\,\epsilon^{ab} e_{ia}\,\rho_b .
\end{aligned} \tag{4}$$

The generic Ricci curvature can be obtained as follows:

$$\begin{aligned}
\mathcal{R} &= \mathcal{R}_{ij} dx^i \wedge dx^j = \mathcal{R}^A X_A = \mathcal{R}_{ij}{}^A X_A dx^i \wedge dx^j, \\
\mathcal{R}_{ij} &= \partial_{[i} h_{j]} + [h_i, h_j] = \mathcal{R}_{ij}{}^A X_A = T_{ij}{}^a\ P_a + R_{ij}\ J + F_{ij}\ Z\ ,
\end{aligned} \tag{5}$$

where the torsion $T_{ij}{}^a$, standard Riemannian curvature R_{ij} and the new gauge field strength F_{ij} have the following forms:

$$\begin{aligned}
T_{ij}{}^a &= \partial_{[i}\, e_{j]}{}^a + \epsilon^{ab}(e_{ib}\,\omega_j - e_{jb}\,\omega_i) - \frac{\Lambda}{k}\,\epsilon^{ab}(e_{ib}\,A_j - e_{jb}\,A_i), \\
R_{ij} &= \partial_{[i}\,\omega_{j]}, \\
F_{ij} &= \partial_{[i}\,A_{j]} + k\,\epsilon^{ab} e_{ia}\,e_{jb}.
\end{aligned} \tag{6}$$

Now, one can write the gauge invariant action as [10]

$$I = \frac{1}{2}\int \eta_A \mathcal{R}^A = \frac{1}{2}\int d^2x\,\epsilon^{ij}\,\eta_A\,\mathcal{R}_{ij}{}^A \tag{7}$$

$$= \frac{1}{2}\int d^2x\,\epsilon^{ij}\,(\eta_a\,T_{ij}{}^a + \eta_2\,R_{ij} + \eta_3\,F_{ij}), \tag{8}$$

where $\eta_A = (\eta_a, \eta_2, \eta_3)$ are the Lagrange multiplier-like fields. Now, using (6), one can rewrite this action in the following form:

$$I = \int d^2x\,\epsilon^{ij}\,\Big\{\eta_a\,\Big(\partial_i e_j{}^a + \epsilon^{ab}\,e_{ib}\,(\omega_j - \frac{\Lambda}{k}\,A_j)\Big) + \eta_2\,\partial_i\omega_j + \eta_3\,\Big(\partial_i A_j + \frac{1}{2}k\,\epsilon^{ab}\,e_{ia}\,e_{jb}\Big)\Big\}. \tag{9}$$

This action is invariant under the gauge transformations (4) and the following transformations of the fields η_a, η_2 and η_3:

$$\begin{aligned}
\delta\eta_a &= k\,\epsilon_a{}^b\,\eta_3\,\rho_b - \epsilon_a{}^b\,\eta_b(\tau - \frac{\Lambda}{k}\,\lambda), \\
\delta\eta_2 &= -\epsilon^{ab}\eta_a\,\rho_b, \\
\delta\eta_3 &= \frac{\Lambda}{k}\,\epsilon^{ab}\eta_a\,\rho_b.
\end{aligned} \tag{10}$$

Now, we will show that the model (9) is dual to the $(1+1)$-dimensional *AdS* gravity. We know that $SO(2,1)$ gauge symmetric gravity action can be obtained by use of the following algebra (anti de Sitter algebra for $k' \neq 0$):

$$[J, P_a] = \epsilon_{ab} P^b, \qquad [P_a, P_b] = k'\epsilon_{ab} J, \tag{11}$$

as follows [10]:

$$\tilde{I} = \int d^2x\,\epsilon^{ij}\,\Big\{\tilde{\eta}_a\,\Big(\partial_i e_j{}^a + \epsilon^{ab}\,e_{ib}\,\omega_j\Big) + \tilde{\eta}_2\,\Big(\partial_i\omega_j + \frac{1}{2}k'\,\epsilon^{ab}\,e_{ia}\,e_{jb}\Big)\Big\}. \tag{12}$$

An $SO(2,1)$ invariant action for two-dimensional gravity was first constructed in [3] where the aim was to reconstruct the proposed two-dimensional Einstein equation from a two-dimensional gauge theory of gravity. Although the notation adopted in [3] is different from our notation,[4] but it can be shown that the action constructed in [3] is equivalent to the $(1+1)$-dimensional *AdS* gravity model (12). Now, by selecting $\eta_3 = -\frac{\Lambda}{k}\eta_2$ in our model (9), it is dual (canonically transformed) to the *AdS* gravity (12); i.e. the following map:

$$\begin{aligned}
&\omega_i \longrightarrow \omega_i - \frac{\Lambda}{k}A_i, \qquad e_i{}^a \longrightarrow e_i{}^a, \qquad \tilde{\eta}_a \longrightarrow \eta_a, \\
&\tilde{\eta}_2 \longrightarrow \eta_2, \qquad k' = -\Lambda,
\end{aligned} \tag{13}$$

transforms the AdS_2 gravity model (12) to our model (9). In the following, we will show that this map is a canonical one. The canonical Poisson-brackets and the Hamiltonian related to the AdS_2 gravity model (12) are as follows:

$$\begin{aligned}
&\{(\tilde{\Pi}_e)_i{}^a(x)\ ,\ e_j{}^b(y)\} = \epsilon_{ij}\eta^{ab}\delta(x-y), \\
&\{(\tilde{\Pi}_\omega)_i(x)\ ,\ \omega_j(y)\} = \epsilon_{ij}\delta(x-y), \\
&\{(\tilde{\Pi}_{\tilde{\eta}_a})^a(x)\ ,\ \tilde{\eta}^b(y)\} = \eta^{ab}\delta(x-y), \\
&\{(\tilde{\Pi}_{\tilde{\eta}_2})(x)\ ,\ \tilde{\eta}_2(y)\} = \delta(x-y),
\end{aligned}$$

[1] This algebra is isomorphic to the four-dimensional Lie algebra $\mathcal{A}_{3,8} \oplus \mathcal{A}_1$ [28].

[2] Note that the commutation relation $[J, Z] = 0$ can be obtained from the Jacobi identity $[J, [P_a, P_b]] + \text{cyclic terms} = 0$.

[3] Centrally extended Poincaré algebra (or Maxwell algebra) in $1+1$ dimensions is isomorphic to the four-dimensional Lie algebra $\mathcal{A}_{4,8}$ [28].

[4] The $SO(2,1)$ invariant action for two-dimensional gravity constructed in [3] is $\frac{1}{2}\int \epsilon^{ABC} R_{AB}\ \phi_C$ where $R_{AB} = d\omega_{AB} - \omega_{AC}\ \omega^C{}_B$ and $A, B = 0,1,2$. The field ω_{AB} contains both the spin connection ω_{ab} and vierbein e_a where $a, b = 0,1$ such that we have $\omega_{a2} = \ell^{-1} e_a$. Using the field redefinitions $\phi_0 \equiv \ell\tilde{\eta}_1$, $\phi_1 \equiv \ell\tilde{\eta}_0$, $\phi_2 \equiv \tilde{\eta}_2$ and $\omega_{01} \equiv \omega$, one can easily show that this action is equivalent to the AdS_2 gravity action (12) with $k' = \ell^{-2}$.

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