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Generalized multi-Proca fields

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ABSTRACT

We extend previous results on healthy derivative self-interactions for a Proca field to the case of a set of massive vector fields. We obtain non-gauge invariant derivative self-interactions for the vector fields that maintain the appropriate number of propagating degrees of freedom. In view of the potential cosmological applications, we restrict to interactions with an internal rotational symmetry. We provide a systematical construction order by order in derivatives of the fields and making use of the antisymmetric Levi-Civita tensor. We then compare with the one single vector field case and show that the interactions can be broadly divided into two groups, namely the ones obtained from a direct extension of the generalized Proca terms and genuine multi-Proca interactions with no correspondence in the single Proca case. We also discuss the curved spacetime version of the interactions to include the necessary non-minimal couplings to gravity. Finally, we explore the cosmological applications and show that there are three different vector field configurations giving rise to isotropic solutions. Two of them have already been considered in the literature and the third one, representing a combination of the first two, is new and offers unexplored cosmological scenarios.

1. Introduction

The accelerated expansion of the universe discovered almost two decades ago still remains a challenging puzzle for modern cosmology. Assuming General Relativity as the appropriate theory describing the gravitational interaction on cosmological scales, the cosmic acceleration can be accounted for by simply including a cosmological constant. However, its required value in agreement with observations turns out to be tiny as compared to the expected *natural* value and this discord puts on trial our theoretical understanding of gravity and the standard techniques of quantum field theory [1]. This problem has triggered a plethora of attempts to modify gravity on large scales and most of them eventually unfold in the form of additional scalar fields, which can then be used as a condensate whose energy density can drive the accelerated expansion of the universe. Some modified gravity scenarios resorted to braneworld models with extra-dimensions as possible mechanisms to generate acceleration and/or alleviate the hierarchy problem, being the DGP model [2] a paradigmatic example. In this model, the effective scalar field describing the vibrations of the brane presents interesting features among which we can mention interactions involving second derivatives of the scalar field, which nevertheless lead to second order field equations so that the Ostrogradski instability is avoided, and a Galilean symmetry allowing for constant shift not only in the field itself, but also in its gradient. These properties were then generalized to find the most general Lagrangian sharing such features [3] and are known as Galileon interactions. A nice property of these interactions is their radiative stability under quantum corrections [4], even if they fail to tackle the cosmological constant problem. The covariantization of these Galileon interactions to include gravity requires the introduction of non-minimal couplings in order to maintain the second order nature of the equations of motion and this led to the rediscovery of the now so-called Horndeski Lagrangians, which are the most general scalar–tensor theories leading to second order equations of motion [5]. Further developments showed that it is in fact possible to build more general scalar–tensor theories with higher order equations of motion, but still without additional propagating degrees of freedom and, therefore, avoiding the Ostrogradski instability [6].

Along the lines of constructing consistent theories for scalar–tensor interactions, one can try to build analogous consistent theories for a vector field. Interestingly, very much like the Galileon interactions can be elegantly obtained from geometrical construc-

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tions in higher dimensions [7,8], it is possible to obtain vector Galileon interactions within the framework of (generalized) Weyl geometries [9,10]. For a massive vector field one can indeed construct non-gauge invariant derivative self-interactions of the vector field with the requirement that only three degrees of freedom propagate, as it corresponds to a massive vector field. The resulting theory is composed by the generalized Proca interactions which guarantee having second order equations of motion and the desired 3 polarizations for the vector [11,12]. Not surprisingly and in a similar way to the Horndeski Lagrangians for a scalar field, the generalized Proca interactions can also be further extended to the case of more general vector–tensor interactions with higher order equations of motion, but still propagating three polarizations [13].

The goal of this work is to extend the generalized Proca interactions to the case of several interacting vector fields to obtain a multi-Proca version of the healthy non-gauge invariant derivative interactions. A similar extension has also been pursued for the case of scalar Galileon interactions, resulting in the so-called multi-Galileon theories [14]. We will apply a construction scheme taking advantage of the symmetries of the Levi-Civita tensor in the same spirit as the one applied to the single Proca field in [12]. For this purpose, we will go order by order in derivatives of the vector fields and construct the interactions to guarantee that the temporal components of the vector fields do not propagate and, thus, giving rise to healthy interactions. Interactions for a generalized $SU(2)$ Proca field have been considered in [15,16]. In [16] the study was limited to interactions with up to six Lorentz indices using a different approach. At the coincident orders, our interactions agree with theirs. Furthermore, our different systematical procedure allows us to construct interactions which are beyond the orders considered in [16].

The paper is organized as follows. We will start by very briefly reviewing non-abelian gauge theories. In section 3 we will proceed to the systematical construction of the interactions, ending with a summary of all the interactions. Section 5 will be devoted to making a comparison between the obtained interactions and those present in the single vector field case. Furthermore, this will allow us to identify some of the required non-minimal couplings to extend the interactions to curved spacetime. Finally, in section 6 we will discuss the possible configurations for the vector fields that will allow for isotropic cosmological solutions and illustrate it with a simple example. In section 7 we discuss our main findings.

Internal group and Lorentz (spacetime) indices will be denoted by Latin $a, b, c, \dots$ and Greek $\alpha, \beta, \gamma, \dots$ letters respectively. We will use the mostly plus signature for the spacetime metric. The dual of an antisymmetric tensor $F_{\mu\nu}$ is defined as $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. We define symmetrization and antisymmetrization as $T_{(\mu\nu)} = T_{\mu\nu} + T_{\nu\mu}$ and $T_{[\mu\nu]} = T_{\mu\nu} - T_{\nu\mu}$ respectively.

2. Non-abelian gauge field

Before proceeding to the construction of the derivative self-interactions for a set of massive vector fields, it will be convenient to briefly review the properties of interacting massless vector fields. It is known that consistency of the interactions for the massless vector fields leads to the full non-abelian gauge structure of Yang–Mills theories. Alternatively, one can start with the Lagrangian for a set of massless vector fields A^a_μ

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{ab}F^{a\mu\nu}F^b_{\mu\nu} \tag{1}$$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ and $\mathcal{G}_{ab}$ a metric in the field space. The isometry group of this metric leads to the presence of global symmetries that, through Noether theorem, gives rise to a set of conserved currents. Then, when the interactions for the fields are introduced as consistent couplings to the currents, again the resulting interactions are given by the Yang–Mills Lagrangian

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{ab}\mathcal{F}^{a\mu\nu}\mathcal{F}^b_{\mu\nu} \tag{2}$$

with the non-abelian field strength

$$\mathcal{F}^a_{\mu\nu} = F^a_{\mu\nu} + gf^{abc}A^b_\mu A^c_\nu \tag{3}$$

where g is the coupling constant of the non-abelian field and f^{abc} are the structure constants of the Lie algebra of the isometry group of $\mathcal{G}_{ab}$ whose generators T_a then satisfy $[T_a, T_b] = if_{ab}{}^cT_c$ and can be normalized so that $\mathrm{Tr}(T_aT_b) = \mathcal{G}_{ab}$, i.e., $\mathcal{G}_{ab}$ is nothing but the corresponding Killing metric of the group. The vector fields then take values in the Lie algebra of the group so that under an isometry transformation with parameters θ^a the vectors transform in the adjoint representation $A^a_\mu \to A^a_\mu + f_{bc}{}^a\theta^bA^c_\mu - \partial_\mu\theta^a/g$, i.e., as it corresponds to a connection. One can then introduce the covariant derivative $D_\mu \equiv \partial_\mu\mathbb{1} - igA^a_\mu T_a$, whose commutator gives, as usual, the curvature: $[D_\mu, D_\nu] = -ig\mathcal{F}^a_{\mu\nu}T_a$. The field strength transforms covariantly[1] and, thus, the above Lagrangian is gauge invariant. Adding a mass term for the vector fields breaks the non-abelian gauge symmetry. This can be done either by adding a hard mass term to the Lagrangian or through a Higgs mechanism so that the gauge symmetry is spontaneously broken and it is non-linearly realized. Either way, the resulting Lagrangian will read

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{ab}\mathcal{F}^{a\mu\nu}\mathcal{F}^b_{\mu\nu} - \frac{1}{2}M_{ab}A^a_\mu A^{b\mu}, \tag{4}$$

with M_{ab} the mass matrix. Although the gauge symmetry is broken by the mass term, the original global symmetry can remain if, for instance, $M_{ab} \propto \mathcal{G}_{ab}$. Our main goal in this work is to construct the generalization of the massive non-abelian vector field to include derivative self-interactions. For the construction we will follow closely the approach applied in [12]. For the sake of concreteness, we will restrict our analysis to the case of an internal rotational group for the vector fields, which can be viewed as a descendant of an original $SU(2)$ gauge symmetry. In that case, we have that the Killing metric is the Euclidean metric δ_{ab} (so lowering and raising group indices will be innocuous operations) and the structure constants are given by the completely antisymmetric Levi-Civita symbol ϵ_{abc}. Furthermore, we will assume that the global symmetry remains so that the number of possible interactions is substantially reduced. Since for $SO(3)$ the adjoint and the fundamental representations are equivalent, we will no make any distinction in the following.

3. Systematical construction

In this section we will systematically construct the healthy derivate self-interactions for a set of vector fields with an internal global $SO(3)$ symmetry, as explained above. The procedure that we will follow is then based in the usual construction making use of the antisymmetry of the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$ and that has been extensively exploited in the literature to construct healthy interactions. In particular, it was used in [17] to generalize the Galileon interactions to the case of arbitrary p-forms. For a set of interacting 1-forms (resembling the case under study here) it is possible to write Galileon interactions while retaining a non-abelian gauge symmetry. However, the first dimension where they are non-trivial is $D = 5$ and, since our analysis will be performed

[1] As opposed to the abelian case where the field strength is gauge invariant.