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On the gravitational instability in the Newtonian limit of MOG

Fatimah Shojai [a,b,*], Samira Cheraghchi [a], Hamed Bouzari Nezhad [a]

[a] Department of Physics, University of Tehran, Tehran, Iran
[b] Foundations of Physics Group, School of Physics, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran

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ABSTRACT

We have found some analytical cosmological solutions to MOdified Gravity (MOG). These solutions describe different evolutionary epochs of an isotropic and homogeneous universe. During each epoch, the evolution of cosmological perturbation is studied in the Newtonian framework and compared with the corresponding results of GR.

1. Introduction

There are many observations in cosmology leading to the introduction of mysterious aspects of dark matter and dark energy. One can think of an alternative gravitational dynamics instead of introducing dark matter and dark energy. At present, many modified gravitational theories are proposed [1]. Two usual approaches are adding higher order curvature invariants and extra fields in the gravitational action. For the latter case, two important extended theories are Tensor–Vector–Scalar (TeVeS) theory [2] and MOdified Gravity (MOG) [3]. In both, gravity is described by some tensor, vector and scalar fields and there exist some free parameters which must be fitted to observations.

MOG introduced in order to explain the flat rotation curves of spiral galaxies and mass discrepancy in galaxy clusters without the need of exotic dark matter [4] as well as explaining the large scale structure of the universe [5]. In this article we study how small initial inhomogeneities grow in an expanding universe in MOG. We take the Newtonian viewpoint which is an adequate description of relativistic treatment on sub-horizon scale and for non-relativistic matter perturbation. To do this, it is required to have the background metric of space–time. Thus we must first find some cosmological solutions of MOG. Some of these solutions are obtained in [6] using the numerical methods and in [7] via the Noether symmetry approach [8].

In this paper, after reviewing the basic equations of MOG, we derive some cosmological solutions for a spatially flat Friedmann–Robertson–Walker (FRW) universe with a perfect fluid in section 3. The first one corresponds to an exact power-law evolution of dynamical fields while the other one corresponds to a universe which is dominated by a single component fluid together with G-field. We consider G-radiation, G-phion and G-matter dominated universes. These are interested since we want to study the evolution of inhomogeneity in these epochs. A brief discussion of Newtonian analysis of gravitational instability in MOG [9] is presented in section 4. Then we study how the sub-Hubble fluctuations evolve in an expanding universe and compare the result with standard cosmology in section 5.

2. The cosmological field equations of MOG

MOG theory postulates more gravitational fields than GR. In addition to metric tensor, $g_{\alpha\beta}$, there are a massive Proca vector field $\phi_\mu(x)$ which is coupled to matter and two additional scalar fields, $\mu(x)$, the mass of the vector field, and $G(x)$, a variable gravitational constant. The action of this theory can be written as [3]:

$$S = S_{grav} + S_\Lambda + S_\phi + S_\mu + S_G + S_M \tag{1}$$

where S_M, is the matter sector of action and S_{grav}, S_Λ, S_ϕ, S_μ and S_G are given by:

$$S_{grav} = \frac{1}{16\pi}\int \sqrt{-g}\frac{1}{G}R d^4x \tag{2}$$

$$S_\Lambda = -\frac{1}{8\pi}\int \sqrt{-g}\frac{1}{G}\Lambda d^4x \tag{3}$$

$$S_\phi = -\int \sqrt{-g}\omega_0\left[\frac{1}{4}B_{\alpha\beta}B^{\alpha\beta} + V_\phi\right] d^4x \tag{4}$$

$$S_\mu = \int \sqrt{-g}\frac{1}{G\mu^2}\left[\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\mu\nabla_\beta\mu - V_\mu\right] d^4x \tag{5}$$

$$S_G = \int \sqrt{-g}\frac{1}{G^3}\left[\frac{1}{2}g^{\alpha\beta}\nabla_\alpha G\nabla_\beta G - V_G\right] d^4x \tag{6}$$

* Corresponding author.
E-mail address: fshojai@ut.ac.ir (F. Shojai).

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in which ω_0 is a dimensionless positive coupling constant and $B_{\alpha\beta} = \nabla_\alpha \phi_\beta - \nabla_\beta \phi_\alpha$. R is the Ricci scalar and Λ is the cosmological constant. V_ϕ, V_μ and V_G are self interaction potentials of the vector and scalar fields. The constant coupling parameter ω_0 in the action and the resulting field equations plays no significant role and can be absorbed in other variables, and thus hereafter we set it equal to unity. Considering a spatially flat FRW metric:

$$dS^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right) \tag{7}$$

in which $a(t)$ is the scale factor of the universe. Here we shall use the original MOG potential in the form [3]:

$$V_\phi = -\frac{1}{2}\mu^2 \phi_\mu \phi^\mu \tag{8}$$

and set the other potential functions to zero. The MOG cosmological equations are:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} + \frac{\dot{G}}{G}\frac{\dot{a}}{a} - \frac{1}{12}\frac{\dot{\mu}^2}{\mu^2} - \frac{1}{24}\frac{\dot{G}^2}{G^2} + \frac{G}{12}\mu^2\phi_0^2 \tag{9}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} + \frac{1}{2}\frac{\dot{G}}{G}\frac{\dot{a}}{a} + \frac{1}{6}\frac{\dot{\mu}^2}{\mu^2} + \frac{1}{2}\frac{\ddot{G}}{G} - \frac{11}{12}\frac{\dot{G}^2}{G^2} - \frac{G}{6}\mu^2\phi_0^2 \tag{10}$$

$$\frac{\ddot{G}}{G} = 32\pi G\rho + 12\frac{\ddot{a}}{a} + 9\frac{\dot{G}}{G}\frac{\dot{a}}{a} - 2\frac{\dot{\mu}^2}{\mu^2} + \frac{\dot{G}^2}{G^2} + G\mu^2\phi_0^2 \tag{11}$$

$$\frac{\ddot{\mu}}{\mu} = \frac{\dot{\mu}^2}{\mu^2} - 3\frac{\dot{\mu}}{\mu}\frac{\dot{a}}{a} + \frac{\dot{G}}{G}\frac{\dot{\mu}}{\mu} - G\mu\frac{\partial V_\phi}{\partial \mu} \tag{12}$$

$$\frac{\partial V_\phi}{\partial \phi_0} = 16\pi\kappa\rho \tag{13}$$

where κ is a coupling constant that appears in variation of matter action with respect to ϕ_α [3] and a dot denotes derivative with respect to the cosmic time. The zeroth component of the vector field is the only non-zero component of ϕ-field because of the cosmological principle which also implies the conservation of energy-momentum tensor of cosmological fluid in MOG theory [10]. The above equations are generalized Friedmann equations and the equations of motion for G and μ fields respectively. The last relation gives the coupling of matter to ϕ field which is neutral and doesn't couple to photons. Therefore, ϕ field perturbations can grow during the radiation dominated era in which baryons and photons are strongly coupled.

Using the definition of the energy-momentum tensor of fields (the index f can be Λ, ϕ, μ or G).

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_f}{\delta g^{\mu\nu}} \tag{14}$$

we have:

$$T_{00}^{(\phi)} = \rho^{(\phi)} = \frac{1}{32\pi}\mu^2\phi_0^2 \tag{15}$$

$$T_{00}^{(\mu)} = \rho^{(\mu)} = -\frac{1}{32\pi G}\frac{\dot{\mu}^2}{\mu^2} \tag{16}$$

$$T_{00}^{(G)} = \rho^{(G)} = -\frac{1}{64\pi G}\frac{\dot{G}^2}{G^2} \tag{17}$$

$$\rho^{(\Lambda)} = \frac{\Lambda}{8\pi G} \tag{18}$$

Using the above relations, one can rewrite the first two equations of motion, (9) and (10), as follows:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\rho + \rho^{(\Lambda)} + \rho^{(\phi)} + \rho^{(\mu)} + \rho^{(G)}\right) + \frac{\dot{G}}{G}\frac{\dot{a}}{a} \tag{19}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\Big[(\rho + 3p) + \rho^{(\Lambda)}(1 + 3\omega_\Lambda) + \rho^{(\phi)}(1 + 3\omega_\phi) + \rho^{(\mu)}(1 + 3\omega_\mu) + \rho^{(G)}(1 + 3\omega_G)\Big] + \frac{1}{2}\frac{\ddot{G}}{G} + \frac{1}{2}\frac{\dot{G}}{G}\frac{\dot{a}}{a} - \frac{\dot{G}^2}{G^2} - \frac{1}{8}G\mu^2\phi_0^2 \tag{20}$$

where $\omega_\Lambda = -1$, $\omega_\mu = \omega_G = 1$, $\omega_\phi = 0$. We use this form of equations in the next section in which we study how a single or two fluid components drive the evolution of the universe.

To obtain a closed system of equations, one must specify one equation of state for matter which is usually assume to be linear in cosmology:

$$p = \omega\rho \tag{21}$$

where the equation of state parameter ω is a constant.

Before discussing some cosmological solutions of MOG theory, let us explain briefly how this theory is consistent with the observational data. Below, we list some of the more important ones:

- Big-Bang nucleosynthesis
  At the nucleosynthesis era, the gravitational constant is very close to the Newtonian one [5] and thus the abundances of the light elements agree with the available data.
- Rotation curve
  Using the weak field approximation of MOG [11,12], one obtains an effective gravitational potential which has two free parameters, $\alpha = (G_\infty - G_N)/G_N$ and $\tilde{\mu}$. G_N is the Newtonian gravitational constant, G_∞ is the effective gravitational constant at infinity and $\tilde{\mu}$ is the mass of vector field which is constant at this approximation. Best fitting of the galaxy rotation curves gives: $\alpha \simeq 8.89 \pm 0.34$, $\tilde{\mu} \simeq 0.042 \pm 0.004$ kpc^{-1} [11,12].
- Baryon acoustic oscillations
  Since the Jeans length of pressureless phion particles (the particle of ϕ field) is very small, there is no oscillatory behaviour for these particles. Their perturbations grow while baryon perturbations oscillate before decoupling [5].
- The CMB power spectrum
  In the present universe: $(\Omega_b^0)_{MOG} = (\Omega_b^0)_{\Lambda CDM}$, $\rho_\phi^0 \ll \rho_b^0$, $G^0 = G_N(1+\alpha)$ and $(G_N\rho)_{\Lambda CDM} = (G_N(1+\alpha)\rho)_{MOG}$ where $\rho_{\Lambda CDM} = \rho_b + \rho_{CDM}$ and $\rho_{MOG} = \rho_b$. Going back to the past, at the decoupling time, $\rho_\phi \gg \rho_b$ and $\alpha \ll 1$, thus $(G_N\rho)_{\Lambda CDM} = (G_N\rho_\phi)_{MOG}$ [5,13]. This shows that the CMB power spectrum in MOG agrees with the corresponding one in ΛCDM model.

3. Some exact cosmological solution of MOG

In this section we find some cosmological solutions of MOG theory with zero cosmological constant. The simplest analytical solutions are power-law type. Assuming that:

$$a(t) = (\frac{t}{t_0})^\lambda, \qquad G(t) = G_0(\frac{t}{t_0})^\sigma, \qquad \rho(t) = \rho_0(\frac{t}{t_0})^{-3(1+\omega)\lambda},$$
$$\mu(t) = \mu_0(\frac{t}{t_0})^\alpha, \qquad \phi_0(t) = \tilde{\phi}_0(\frac{t}{t_0})^\beta \tag{22}$$

where a subscript "0" indicates the value of any quantity evaluated at the present and the usual normalization $a_0 = 1$ is used. Inserting (22) into equations (7)–(12), we find that