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Reconciling threshold and subthreshold expansions for pion–nucleon scattering

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ABSTRACT

Heavy-baryon chiral perturbation theory (ChPT) at one loop fails in relating the pion–nucleon amplitude in the physical region and for subthreshold kinematics due to loop effects enhanced by large low-energy constants. Studying the chiral convergence of threshold and subthreshold parameters up to fourth order in the small-scale expansion, we address the question to what extent this tension can be mitigated by including the $\Delta(1232)$ as an explicit degree of freedom and/or using a covariant formulation of baryon ChPT. We find that the inclusion of the Δ indeed reduces the low-energy constants to more natural values and thereby improves consistency between threshold and subthreshold kinematics. In addition, even in the Δ-less theory the resummation of $1/m_N$ corrections in the covariant scheme improves the results markedly over the heavy-baryon formulation, in line with previous observations in the single-baryon sector of ChPT that so far have evaded a profound theoretical explanation.

1. Introduction

The approximate chiral symmetry of QCD imposes strong constraints on low-energy hadron dynamics, which can be explored systematically in the framework of chiral perturbation theory (ChPT) [1–3]. While in the meson sector the expansion proceeds directly in terms of momenta and quark masses divided by a breakdown scale Λ_b, typically identified with the mass of the $\rho(770)$ or the scale of chiral symmetry breaking $\Lambda_\chi = 4\pi F_\pi \sim 1.2\,\text{GeV}$, in the baryon sector the nucleon mass m_N represents a new scale that needs to be taken into account in order not to spoil the chiral power counting [4]. Heavy-baryon ChPT (HBChPT) [5,6] achieves this by systematically expanding the effective Lagrangian in $1/m_N$, identifying $\Lambda_b \sim m_N$. In subsequent years, several variants of covariant baryon ChPT have been developed [7–12], in which the power-counting-violating part is subtracted in one way or another. While originally motivated by the desire to preserve the analytic structure of the amplitude in the vicinity of anomalous thresholds and unitarity cuts, it has also been observed that the resummation of $1/m_N$ corrections can improve the phenomenology even in kinematic regions where the HB formulation does reproduce the analytic structure correctly [13–16].

The efficacy of different formulations of baryon ChPT has implications beyond the single-nucleon sector. In chiral effective field theory, the extension of ChPT to multi-nucleon systems [17–21], the low-energy constants (LECs) that appear in pion–nucleon (πN) scattering determine the long-range part of the nucleon–nucleon (NN) potential as well as three-nucleon forces. While the use of the HB formulation is common to all implementations to date, $1/m_N$ corrections are often counted suppressed by one additional order compared to the standard single-nucleon HB counting, to account for the fact that the breakdown scale in the multi-nucleon sector tends to be lower than in single-nucleon applications [18,20] (this counting scheme will be referred to as HB-NN counting in the following, in contrast to the standard HB-πN).

Recently, the combination of dispersion theory in the form of Roy–Steiner (RS) equations [22–28] with precision measurements of the πN scattering lengths in pionic atoms [29–33] resulted in a reliable representation of the πN scattering amplitude in

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the whole low-energy region, both in the physical region and for subthreshold kinematics. Surprisingly, the matching to HBChPT revealed that, in contrast, the chiral representation is not accurate enough to relate these two regions [25]. These findings can be best illustrated considering the parameters in the expansion around threshold and the subthreshold point: with LECs determined in the subthreshold region, where due to the absence of unitarity cuts ChPT is expected to converge best [34], the chiral series fails to reproduce some of the threshold parameters. The reason for this behavior can be traced back to loop diagrams producing terms that scale as $g_A^2(c_3-c_4)\sim -16\,\text{GeV}^{-1}$, an enhancement that is, at least partially, generated by saturation of the LECs c_i with the $\Delta(1232)$ resonance. As argued in [25], this inconsistency between subthreshold and threshold kinematics implies that in a HB formulation, LECs determined at the subthreshold point are preferable for multi-nucleon applications, given that the kinematics for the two-pion exchange in the NN potential are much closer to the subthreshold point than to the physical region in πN scattering.

In this paper we address the question to what extent consistency between subthreshold and physical region can be restored by introducing the Δ as an explicit degree of freedom, and/or by using a covariant formulation of baryon ChPT. The Δ is included within the small-scale expansion [35], counting the difference $\varepsilon = m_\Delta - m_N$ in the same way as a momentum scale p. πN scattering with explicit Δ degrees of freedom has been considered before at $\mathcal{O}(\varepsilon^3)$ in HB [36] and covariant [37] formulations, as well as within the δ-counting of [38] up to $\mathcal{O}(p^3)$ in a covariant scheme [15] (see also [39]). Here, we extend the analysis to full one-loop order $\mathcal{O}(\varepsilon^4)$ and study the predictions for the leading eight threshold parameters, with LECs determined from the subthreshold parameters predicted by the RS analysis [26]. After a brief introduction to the formalism in Sect. 2, we first present the results when including the Δ in HBChPT in Sect. 3, and then extend the analysis towards a covariant formulation in Sect. 4. We offer our conclusions in Sect. 5. Details on large-N_c constraints and correlation coefficients of the extracted LECs are summarized in the appendices.

2. Formalism

For the calculation of the threshold and subthreshold parameters, we heavily rely on the full $\mathcal{O}(\varepsilon^4)$ results from [40], where the T-matrix for the process $\pi N\to\pi N$ is calculated in the small-scale expansion

$$\varepsilon = \left\{\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{m_\Delta - m_N}{\Lambda_b}\right\} \qquad \text{with} \qquad \Lambda_b \in \{\Lambda_\chi, m_N\}, \tag{1}$$

in the HB as well as in the covariant approach. The standard on-mass-shell renormalization scheme is employed for the leading-order LECs, where pion, nucleon, and Δ masses are denoted by M_π, m_N, and m_Δ, respectively, and the axial couplings of the nucleon and nucleon–Δ transition by g_A and h_A (both axial couplings are renormalized at the pion vertex instead of the axial current). After absorbing redundant contributions proportional to the LECs d_{18} from $\mathcal{L}^{(3)}_{\pi N}$,[1] $e_{19,20,21,22,36,37,38}$ from $\mathcal{L}^{(4)}_{\pi N}$, $b_{3,6}$ from $\mathcal{L}^{(2)}_{\pi N\Delta}$, c_i^Δ from $\mathcal{L}^{(2)}_{\pi\Delta}$, h_i from $\mathcal{L}^{(3)}_{\pi N\Delta}$, and k_i from $\mathcal{L}^{(4)}_{\pi N\Delta}$, the πN scattering amplitude at $\mathcal{O}(\varepsilon^4)$ depends on the LECs $c_{1,2,3,4}$ from $\mathcal{L}^{(2)}_{\pi N}$, $d_{1+2,3,5,14-15}$ from $\mathcal{L}^{(3)}_{\pi N}$, $e_{14,15,16,17,18}$ from $\mathcal{L}^{(4)}_{\pi N}$, h_A from $\mathcal{L}^{(1)}_{\pi N\Delta}$, g_1 from $\mathcal{L}^{(1)}_{\pi\Delta}$, and $b_{4,5}$ from $\mathcal{L}^{(2)}_{\pi N\Delta}$. In the HB approach, the LECs c_i, d_i, and e_i are renormalized to absorb UV divergent and additional decoupling-breaking pieces. In the covariant approach, the same set of LECs is needed to cancel UV divergences as well as decoupling- and/or power-counting-breaking pieces [16,40]. In particular, both chiral amplitudes are renormalized in such a way that the explicit difference is of higher order only, $\mathcal{O}(\varepsilon^5)$.

Employing the standard subthreshold and threshold expansion of the πN scattering amplitude, we calculate both sets of the respective coefficients (explicit expressions are provided as supplementary material in the form of a MATHEMATICA notebook). Furthermore, we performed a strict chiral expansion of the covariant expressions to check that the HB expressions determined from the HB amplitude are reproduced. In contrast to the Δ-less case, where the 13 leading subthreshold parameters depend on 13 $\pi\pi NN$-LECs, the expressions in the Δ-ful case depend on 4 additional LECs from the Δ sector. Thus, these additional LECs cannot be extracted by the subthreshold matching but further constraints have to be introduced. In particular, we assume the following conservative estimates for those particular LECs

$$\begin{aligned} h_A &= 1.40 \pm 0.05, & b_4 + b_5 &= (0\pm 5)\,\text{GeV}^{-1},\\ g_1 &= 2.32 \pm 0.26, & b_4 - b_5 &= (0\pm 5)\,\text{GeV}^{-1}, \end{aligned} \tag{2}$$

motivated by large-N_c considerations and, in the case of h_A, supplemented by phenomenology, as explained below, where the input from phenomenology allows us to reduce the uncertainty compared to the large-N_c prediction alone.

Given that the contributions proportional to h_A already appear at leading order, its error is most important for the final uncertainty, but our assignment in (2) is still reasonably conservative. It is consistent with the large-N_c prediction, $h_A = 1.37\pm 0.15$ [41,42], the value extracted from the covariant Δ width at full one-loop order $h_A = 1.43\pm 0.02$ [43], and the recent extraction from NN scattering by the Granada group, $h_A = 1.397\pm 0.009$ [44], where the error refers to statistics only. The contribution proportional to g_1 starts at loop level, $\mathcal{O}(\varepsilon^3)$, and its effect on the threshold and subthreshold parameters is much less relevant. The estimate in (2) corresponds to its large-N_c prediction, i.e. $g_1 = 9/5\,g_A$ with an $\mathcal{O}(1/N_c^2)$ error [41,42]. The values of h_A and g_1 are also consistent with constraint from the Δ width recently derived in [45]. Finally, the LECs b_4 and b_5 only contribute at $\mathcal{O}(\varepsilon^4)$, and their impact on our results is almost negligible. The intervals in (2) are based on a large-N_c calculation, which sets their difference and sum as $b_4 - b_5 = 3/(2\sqrt{2})\,c_4$ and $b_4 + b_5 = 2\sqrt{2}/3\,c_{11}^\Delta$, see Appendix A. The value of c_4 in the relation for $b_4 - b_5$ refers to $\mathcal{O}(\varepsilon^2)$, see Table 1, which corresponds to the consistent order of c_4 in the large-N_c relation and also avoids possible correlations with the redundant Δ-LECs absorbed into the c_i at higher orders, leading to an estimate of about $1\,\text{GeV}^{-1}$. In contrast, the unknown LEC appearing in the sum, c_{11}^Δ, proportional to an isotensor contribution, is fixed to zero. Choosing uncertainties generously to cover possible deviations in both cases (e.g. values obtained in $\pi N\to\pi\pi N$ [46]), we simply vary both combinations within $\pm 5\,\text{GeV}^{-1}$. We also checked that taking even larger intervals for these two parameters does not produce any noticeable effect in our results. In addition, we employ the following numerical values for the various LECs and masses entering the leading-order effective Lagrangian: $M_\pi = 139.57\,\text{MeV}$, $F_\pi = 92.2\,\text{MeV}$, $m_N = 938.27\,\text{MeV}$, $m_\Delta = 1232\,\text{MeV}$ [47], and $g_A = 1.289$. The value for g_A includes the Goldberger–Treiman discrepancy parameterized by d_{18}, using a πN coupling constant $g^2/(4\pi) = 13.7$ [33]. We do not study the effects of the uncertainties of those quantities, which are negligible in comparison to the other uncertainties encountered in the calculation.

In the following, we will proceed in close analogy to [25]. The LECs c_i, d_i, and e_i are matched order-by-order to the respective subthreshold parameters, where we employ the values determined

[1] In all Lagrangians, the upper index denotes the chiral order, the lower the particle content. For explicit expressions we refer to [40].