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Equilibration and freeze-out of an expanding gas in a transport approach in a Friedmann–Robertson–Walker metric

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ABSTRACT

Motivated by a recent finding of an exact solution of the relativistic Boltzmann equation in a Friedmann–Robertson–Walker spacetime, we implement this metric into the newly developed transport approach Simulating Many Accelerated Strongly-interacting Hadrons (SMASH). We study the numerical solution of the transport equation and compare it to this exact solution for massless particles. We also compare a different initial condition, for which the transport equation can be independently solved numerically. Very nice agreement is observed in both cases. Having passed these checks for the SMASH code, we study a gas of massive particles within the same spacetime, where the particle decoupling is forced by the Hubble expansion. In this simple scenario we present an analysis of the freeze-out times, as function of the masses and cross sections of the particles. The results might be of interest for their potential application to relativistic heavy-ion collisions, for the characterization of the freeze-out process in terms of hadron properties.

1. Introduction

Kinetic theory [1] has been widely used to study the nonequilibrium evolution of fluids and plasmas, not only for ordinary substances but also in the relativistic domain [2]. For sufficiently dilute systems, the Boltzmann equation (BE) describes how the one-particle distribution function $f(t,\mathbf{x},\mathbf{k})$ relaxes towards equilibrium. Under a general spacetime metric, this equation reads [3]

$$k^\mu \frac{\partial f(t,\mathbf{x},\mathbf{k})}{\partial x^\mu} + \Gamma^i_{\lambda\mu}\, k^\lambda k^\mu \frac{\partial f(t,\mathbf{x},\mathbf{k})}{\partial k^i} = C[f]\,, \tag{1}$$

where $\Gamma^i_{\lambda\mu}$ are the Christoffel symbols and $C[f]$ represents the (nonlinear) collision integral [2].

A non-trivial solution of this equation (aside from the equilibrium distribution) is extremely hard to obtain in this general case. Nevertheless, several approximations can be used to simplify the nonlinear structure of $C[f]$. One of the simplest methods is the relaxation time approximation (RTA) [4], which provides a linearized collision term. In addition, perturbative solutions based on the existence of some small parameter (like the Knudsen number) are also possible e.g. the Chapman–Enskog expansion [5,6]. On the other hand, the BE can also be addressed by pure numerical techniques, known as molecular dynamics simulations or Boltzmann–Uehling–Uhlenbeck (BUU) transport. For relativistic heavy-ion collisions (RHICs), where a mixture of relativistic particles is subjected to mutual interactions and mean-field potentials, the system of coupled BEs can be solved by Monte Carlo methods, as in [7–12]. These numerical approaches—more suitable for these complicated systems—also introduce systematic uncertainties, originating from algorithmic approximations and truncations. For this reason, the finding of exact non-trivial solutions of Eq. (1), at least in particular scenarios, is important to test different methods and approximations, either semianalytical or purely numerical.

In RHICs the dynamics and geometry of the created fireball provide certain degrees of symmetry, from which simplified models have been proposed [13]. For some of them, exact solutions of the BE have been found under the RTA. For example, in the Bjorken model [14] (describing a boost-invariant longitudinal expansion) a semianalytic result has already been calculated by Baym [15]. An exact solution in a Gubser expansion (allowing an additional expansion in the transverse plane) has been obtained in [16]. An exact solution in a 3D conformal expanding medium, or Hubble flow, was recently found in [17,18] under the RTA.

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This last scenario represents an interesting case with applications to RHICs but also in a cosmological context to describe an expanding universe [19–21]. An exact analytical solution of the BE for an expanding medium has recently been presented in [22,23]. This solution is valid for massless particles, interacting in a flat universe under a Friedmann–Robertson–Walker (FRW) metric. The particle–particle interactions are assumed to follow constant total cross sections, but the full nonlinear structure of the collision operator is kept, i.e. no RTA is assumed. The exact solution obtained in [22,23] was used to test a linear approximation of the BE, and approximate solutions based on the RTA.

In this paper we compare a numerical solution of Eq. (1) with this exact result. We employ the new hadronic transport approach SMASH (Simulating Many Accelerated Strongly-interacting Hadrons) [12]. It is used to simulate hot and dense strongly interacting matter with the goal of exploring the quark gluon plasma phase diagram by performing comparisons to experimental data from heavy ion experiments at different accelerators such as the Large Hadron Collider (LHC), the Relativistic Heavy Ion Collider and the SIS-18 at the GSI Helmholtzzentrum für Schwerionenforschung. SMASH constitutes an effective numerical solution of the equations of motion associated with Eq. (1) using a geometrical collision criterion as in UrQMD[1] under a Minkowski metric, i.e. $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$. We adapt the SMASH dynamics to work on an expanding homogeneous, isotropic gas of massless particles with a FRW metric $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$.[2] In this case the BE is reduced to [21,3,23]

$$k^{\mu} u_{\mu} u_{\nu} \partial^{\nu} f(t,k) = C_{gain}[f] - C_{loss}[f] \,, \qquad (2)$$

where the gain and loss terms present their full nonlinear structure.

Our first goal is to present a non-trivial test of the SMASH code in an expanding geometry by comparing our outcome to the exact solution given in [22,23]. We also check our results against the numerical solution of Eq. (2) for a different initial condition given in [23]. Then, we exploit the flexibility of SMASH to solve the transport equation in an expanding system of massive particles, generating a dynamical freeze-out (or decoupling) due to the Hubble expansion. This opens up the possibility to study more realistic systems of interest in cosmological scenarios, or in RHICs.

In Sec. 2 we present the SMASH solution to the Boltzmann equation for massless particles using several initial conditions, in particular the one for which an exact analytical solution is known. In Sec. 3 we introduce a toy model of freeze-out for relativistic particles when the Hubble rate exceeds the interaction rate. We discuss how the freeze-out time can be extracted from the final spectrum of particles. Finally, we present our conclusions and outlook in Sec. 4.

2. SMASH solution of the Boltzmann equation under a FRW spacetime

The authors of Ref. [22,23] have calculated an exact solution of the Boltzmann equation (2) for an infinite gas of massless particles with constant elastic cross-section. This is a very particular system, with symmetry properties that help to simplify the transport equation. This scenario is physically motivated by the expansion of the universe in the radiation-dominated era [19].

SMASH is a recently developed transport approach used to describe the hadronic stage of heavy-ion collisions at low and intermediate energies with applications from GSI to LHC physics. In particular, one can use SMASH to simulate a gas of massless particles with constant elastic cross-section σ. In this work we use a spherical volume filled with N particles (this number remains constant due to the absence of number-changing processes). The particles are initialized with an isotropic, homogeneous spatial distribution according to a given initial condition.

Whilst it appears a formidable task to adapt SMASH to a general spacetime, the FRW metric we wish to implement is fairly simple. As SMASH operates in physical phase-space variables, we will always work with the physical 3-momenta $k = k_{phys}$, as opposed to the approach in [22,23], which uses the covariant momenta.[3] The equations of motion of the particles reflect the physical expansion of the universe. The velocity measured by a comoving observer to the expanding spacetime (sometimes called peculiar velocity) is combined with the Hubble flow: $v^i_{Hubble} = H(t)x^i$, where $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter. The physical momentum of the particle suffers a redshift and scales as $1/a(t)$.

Particle collisions are not affected by the Hubble expansion, because the characteristic collision time is always much smaller than $H^{-1}(t)$, so during the collision the particles do not feel the expansion of the universe.

For a gas of massless particles (radiation) the cosmic scale factor is fixed by the Friedmann equation to be of the form $a(t) \sim t^{\frac{1}{2}}$ [19,21,20]. Following [22,23] we adopt the solution $a(t) = \sqrt{1 + \frac{b_r}{l_0} t}$, where $l_0 = 1/(\sigma n_0)$ is the mean-free path at time $t = 0$ (σ denotes the cross section and n_0 the initial particle density) and b_r is a parameter which contains the density fraction of radiation in the universe and the Hubble parameter itself at $t = 0$.

We initialize the particles in a far-from-equilibrium configuration, according to the momentum distribution in [22,23]

$$f(t=0,k) = \frac{256}{243} \frac{ka}{T_0} \lambda \exp\left(-\frac{4ka}{3T_0}\right) , \qquad (3)$$

where $\lambda = \exp(\mu_0/T_0)$ is the fugacity of the system, and T_0 a parameter, which can be thought of as an initial temperature of the system.[4]

The analytical solution for this initial condition was determined to be

$$f(t,k) = \lambda \frac{e^{-\frac{ka}{\kappa T_0}}}{\kappa^4(\tau)} [4\kappa - 3 + \frac{ka}{\kappa(\tau) T_0} (1 - \kappa(\tau))] \,, \qquad (4)$$

where $\kappa(\tau) = 1 - \exp\left(-\frac{\tau}{6}\right)/4$, and the transformed time variable $\tau = \int_{\hat{t}_0}^{\hat{t}} \frac{1}{a^3(\hat{t}')} d\hat{t}'$ with $\hat{t} = t/l_0$. For the particular form of the $a(t)$ used, we have

$$\tau = \frac{2}{b_r} \left[1 - \left(1 + \frac{b_r}{l_0} t\right)^{-1/2} \right] . \qquad (5)$$

The distribution function is normalized such that

$$N = \int_V \int f(t,k) \frac{d^3k}{(2\pi)^3} d^3\mathbf{x} \,, \qquad (6)$$

where V is the volume of the sphere, and $d^3k = dk k^2 d\Omega_k$.

[1] Ultra-relativistic Quantum Molecular Dynamics.

[2] We only consider the case without spatial curvature K.

[3] The distinction is important in nonorthonormal metrics [20]. In our equations we will always trade the modulus of the covariant momentum—whose magnitude is not modified by the expansion of the system—by the physical one.

[4] Out of equilibrium, the expressions of the particle and energy densities formally coincide with that for an ideal gas, with T_0 playing the role of the temperature.