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11 Dooling with ghost free massive gravity without evolicit square rests $\frac{11}{12}$ Dealing with ghost-free massive gravity without explicit square roots $\frac{76}{77}$ 13 of matrices 78

¹⁵ Alexey Golovnev, Fedor Smirnov

17 82 *Faculty of Physics, St. Petersburg State University, Ulyanovskaya ul., d. 1, Saint Petersburg 198504, Russia*

20 ARTICLE INFO ABSTRACT 85

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₂₂ Article history: **Article history: In this paper we entertain a simple idea that the action of ghost free massive gravity (in metric formula- ₈₇** 23 Received 11 January 2017
23 Received 11 January 2017 Exercived in evised or an 2017 and the elementary symmetric polynomials of the eigenvalues. In particular, we show how one can construct as 25 Available online xxxx
accordite matrix. The method is however absolutely generic. And it also contains the full information on possible 26 Editor: A. Ringwald
1991 - non-standard square roots coming from intrinsic non-uniqueness of the procedure. In passing, we men-⁹² 127 5 tion some hard problems of those apocryphal square roots in the standard approach which might be ⁹³ 93 93 better tackled with our method. The details of the latter are however deferred to a separate paper. 29 2017 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license 24 the quadratic action around Minkowski spacetime without ever taking the square root of the perturbed

30 30 30 $(\text{http://creativecommons.org/licenses/by/4.0/">\%})$ furthp://creativecommons.org/licenses/by/4.0/). Funded by SCOAP³.

1. Introduction

³⁸ with experimental data all over a wide variety of scales. However, class of real matrices) and uniqueness, see also [15,16]. In this pa-³⁹ in the realm of cosmology we have a number of uneasy points per we present a method of dealing with massive gravity without 104 ⁴⁰ including the origin of Dark Energy and the nature of Dark Mat-
explicitly taking the square root of the matrix. In Section 2 we ¹⁰⁵ ⁴¹ ter. It gave rise to a plenitude of attempts to formulate a viable describe the action of massive gravity and its second order expan-⁴² infrared modification of gravity which would hopefully do better sion around flat space. In Section 3 we introduce the formalism 107 ⁴³ in cosmology than GR. In particular, one of such directions which of elementary symmetric polynomials of the eigenvalues, and also 108 ⁴⁴ recently became very popular hinges upon giving a mass to the explain the problems with non-standard square roots in the usual 108 graviton.

⁴⁶ The early days of massive gravity witnessed an almost detec- around flat space. Finally, in Section 5 we conclude. ⁴⁷ tive story which starts from the original paper by Fierz and Pauli **112** time is a starting of the story which starts from the original paper by Fierz and Pauli ⁴⁸ [\[1\]](#page--1-0) which presented the linearised ghost-free massive deformation **2** Massive pravity **113** 113 ⁴⁹ around flat space, and goes through infamous vDVZ discontinuity and all all the series of t ⁵⁰ [\[2,3\]](#page--1-0) of its massless limit, to the potential resolution via Vainshtein We consider the action of massive gravity in the following 51 mechanism [\[4,5\],](#page--1-0) and almost simultaneously to the claim of un-
 62 52 avoidable reappearance of the ghost at non-linear level [\[6\],](#page--1-0) and 117 ⁵³ finally to the ultimate proposal by de Rham, Gabadadze and Tolley $\begin{pmatrix} 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 &$ ⁵⁴ [7-11]. The model requires an additional (fiducial) metric which $S = \int d^N x \sqrt{-g} \left[R + m^2 \sum \beta_n e_n(\sqrt{g-1} \eta) \right]$ (1) ¹¹⁹ ⁵⁵ can either be Minkowski $\eta_{\mu\nu}$ as in the first papers on the sub-
⁵⁵ can either be Minkowski $\eta_{\mu\nu}$ as in the first papers on the sub-⁵⁶ ject, or can be arbitrary [\[12,13\]](#page--1-0) and even dynamical with its own the set of the set 57 Einstein–Hilbert term [14] thereby producing a full-fledged bimet-
by the spacetime is *N*-dimensional with metric $g_{\mu\nu}$, *R* is its 122 \mathcal{F} 133 \mathcal{F} 123 \mathcal{F} 123 \mathcal{F} scalar curvature, and $e_n(\mathcal{M})$'s are elementary symmetric polyno-Einstein–Hilbert term [\[14\]](#page--1-0) thereby producing a full-fledged bimetric gravity.

(F. Smirnov).

 35 **1. Introduction An ugly feature of the model is that the interaction poten-** 100 36 tial is made of $\sqrt{g^{-1}f}$, the square root of the matrix $g^{\mu\alpha}f_{\alpha\nu}$ ¹⁰¹ ³⁷ The theory of General Relativity enjoys a superb agreement which, strictly speaking, lacks both guaranteed existence (in the 102 ^{[4](#page--1-0)5} graviton. The section 4 we apply our method to quadratic gravity ¹¹⁰ formulation. In Section 4 we apply our method to quadratic gravity which, strictly speaking, lacks both guaranteed existence (in the class of real matrices) and uniqueness, see also [\[15,16\].](#page--1-0) In this paper we present a method of dealing with massive gravity without explicitly taking the square root of the matrix. In Section 2 we describe the action of massive gravity and its second order expansion around flat space. In Section [3](#page--1-0) we introduce the formalism of elementary symmetric polynomials of the eigenvalues, and also explain the problems with non-standard square roots in the usual around flat space. Finally, in Section [5](#page--1-0) we conclude.

2. Massive gravity

We consider the action of massive gravity in the following form:

$$
S = \int d^N x \sqrt{-g} \left(R + m^2 \sum_{n=0}^N \beta_n e_n (\sqrt{g^{-1} \eta}) \right) \tag{1}
$$

59 11 gravity.

59 mials of the eigenvalues $λ_i$ of the matrix M_i^{μ} :

61
$$
E
$$
-mail addresses: agolovnev@yandex.ru (A. Golovnev), sigmar40k@yandex.ru $e_n \equiv \sum_{i_1 < i_2 < ... < i_n} \lambda_{i_1} \lambda_{i_2} \cdots \lambda_{i_n}$ (2) 126

63 128 <http://dx.doi.org/10.1016/j.physletb.2017.04.058>

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2 from to the cosmological constant, while $e_N(\sqrt{g^{-1}\eta}) = \frac{1}{\sqrt{-g}}$ adds $\frac{2}{\sqrt{g}} = \frac{8}{3}$ 1 ⁵ and it would contribute to its own cosmological constant. Terms $\sqrt{1}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{2}$, $\sqrt{4}$, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$, $\sqrt{2}$

 $\frac{7}{2}$ Obviously, these polynomials can be described as coefficients in $\frac{2}{1}$ 1 1 1 1 1 1 1 1 2 $\frac{1}{1}$

$$
\det(\mathcal{M} - \lambda \mathbb{I}) = \prod_{n=1}^{N} (\lambda_i - \lambda) = \sum_{n=0}^{N} (-\lambda)^{N-n} \cdot e_n(\mathcal{M}).
$$
\n(3) Of course, the last expression (13) can also be derived from

16 81 Quadratic approximations to the *βⁱ* terms in the action [\(1\)](#page-0-0) are *^e*1*(*M*)* ⁼ *i λⁱ* = [M] (4)

¹⁸ where [M] stands for the trace of M. In other words, we have $\frac{83}{2}$ 19 a shorthand notation which reads $[{\cal M}] \equiv {\cal M}^{\mu}_{\mu}$, $[{\cal M}]^2 \equiv ({\cal M}^{\mu}_{\mu})^2$, $\qquad \qquad \qquad \qquad$ $\qquad \qquad$ \qquad

$$
\begin{array}{ll}\n\text{22} & e_2(\mathcal{M}) = \sum_{i < j} \lambda_i \lambda_j = \frac{1}{2} \left(\left(\sum_i \lambda_i \right)^2 - \sum_i \lambda_i^2 \right) & \sqrt{-g} \cdot e_2(\sqrt{g^{-1} \eta}) = 6 + \frac{3}{2} h^{\mu}_{\mu} + \frac{1}{8} (h^{\mu}_{\mu})^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(h^3), & \text{38} \\
\text{28} & = \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right). & \text{(16)} \\
\text{29} & = \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right). & \text{(17)} \\
\text{30} & = \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right). & \text{(18)} \\
\text{41} & = \frac{1}{2} h^{\mu}_{\mu} + \frac{1}{2} h^{\mu}_{\mu} - \frac{1}{8} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(h^3), & \text{(19)} \\
\text{52} & = \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right). & \text{(10)} \\
\text{53} & = \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right). & \text{(11)} \\
\text{64} & = \frac{1}{2} h^{\mu}_{\mu} + \frac{1}{8} (h^{\mu}_{\mu})^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(h^3), & \text{(18)} \\
\text{75} & = \frac{1}{2} ([\mathcal{M}]^2 - [\mathcal{M}^2] \end{array}
$$

$$
e_n(\mathcal{M}) = \frac{1}{n} \sum_{i=1}^n (-1)^{i-1} [\mathcal{M}^i] \cdot e_{n-i}(\mathcal{M}),
$$

\n
$$
e_{n-i}(\mathcal{M}) = \frac{1}{n} \sum_{i=1}^n (-1)^{i-1} [\mathcal{M}^i] \cdot e_{n-i}(\mathcal{M}),
$$

\n
$$
e_{n-i}(\mathcal{M}) = \frac{1}{n} \sum_{i=1}^n (-1)^{i-1} [\mathcal{M}^i] \cdot e_{n-i}(\mathcal{M}),
$$

\n
$$
e_{n-i}(\mathcal{M}) = \frac{1}{n} \sum_{i=1}^n (-1)^{i-1} [\mathcal{M}^i] \cdot e_{n-i}(\mathcal{M}),
$$

\n(6) by (14).
\nIn this form, the Fierz-Pauli structure of the potential term is not yet obvious. However, we see that there is a non-vanishing first

with $e_n = 0$ for $n > N$.

34 We will be interested only in the case of $N = 4$, for which we $N = \frac{34}{N}$ and $\frac{99}{N}$ and $\frac{99}{10}$ 35 N and the contract of $M = 1$, the value of $M = 100$ get from (6)

$$
e_3(\mathcal{M}) = \frac{1}{6} \left([\mathcal{M}]^3 - 3[\mathcal{M}][\mathcal{M}^2] + 2[\mathcal{M}^3] \right)
$$

\n
$$
= V(0) + m^2 \left(\frac{1}{2}\beta_0 + \frac{3}{2}\beta_1 + \frac{3}{2}\beta_2 + \frac{1}{2}\beta_3 \right) h^{\mu}_{\mu} + \mathcal{O}(h^2)
$$

\n
$$
= 0
$$

and also

$$
e_4(\mathcal{M}) = \frac{1}{24} \Big([\mathcal{M}]^4 - 6[\mathcal{M}]^2[\mathcal{M}^2] + 3[\mathcal{M}^2]^2
$$

\n
$$
+ 8[\mathcal{M}][\mathcal{M}^3] - 6[\mathcal{M}^4] \Big) = det(\mathcal{M}).
$$
\n(8)

\n(9)

\n(10)

\n(110)

\n(111)

\n(211)

46 The relevant parameters are β_1 , β_2 , and β_3 . The mass parameter *m* Being plugged back into the action it vields the familiar result: 47 corresponds to the mass scale of the graviton if the largest of β_i 's α_i is β_i 's α_i and β_i 's α_i and α_i β_i 's α_i 48 (for $i = 1, 2, 3$) is of order one. m^2 (*ii* m^2 (*iii* m^2) 113

49 In this paper we would be interested in linearised gravity $V(h) - V(0) = \frac{V(h)}{2}(\beta_1 + 2\beta_2 + \beta_3) \cdot (h^{\mu\nu}h_{\mu\nu} - (h^{\mu}_{\mu})^2)$ 50 around Minkowski spacetime, so that we take $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ 8 51 with a small perturbation *h* to the metric. We will raise and lower $+O(h^3)$. 52 the indices of *h* by *η*. And then $h^{\mu\nu}$ gives the linear variation of the linear variation g^{-1} with inversed sign $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + \mathcal{O}(h^2)$, or with a better Note that we followed the usual path. However, these calcula-
tions can be simplified by employing the youl known symmetry. 54 accuracy we have **the simplified by employing the well-known** symmetry ₁₁₉ accuracy we have

$$
g^{\mu\alpha}\eta_{\alpha\nu} = \delta^{\mu}_{\nu} - h^{\mu}_{\nu} + h^{\mu\alpha}h_{\alpha\nu} + \mathcal{O}(h^3).
$$
 (9) that $e_n(\mathcal{M}^{-1})$ is a polynomial of $\frac{1}{\lambda_i}$ which can be obtained from

In the standard approach, the square root matrix $\sqrt{g^{-1}\eta}$ would $\frac{g}{g}$ and $\frac{g}{g}$ and

$$
\sqrt{\mathbb{I} - H} = \mathbb{I} - \frac{1}{2}H - \frac{1}{8}H^2 + \mathcal{O}(H^3)
$$

64 with $H = h - h^2 + \mathcal{O}(h^3)$ give the desired result when substituted which also explains the mysterious disappearance of the $(h^{\mu}_{\mu})^2$ - 129 65 into (4), (5), (7), and (8): the second of the second of the second of term from $\sqrt{-g} \cdot e_3(\sqrt{g^{-1}\eta})$. into (4), (5), (7), and (8):

$$
e_1(\sqrt{g^{-1}\eta}) = 4 - \frac{1}{2}h^{\mu}_{\mu} + \frac{3}{8}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}(h^3),
$$
\n
$$
e_2 \text{ (10)} \quad \text{for the cosmological constant, while } e_N(\sqrt{g^{-1}\eta}) = \frac{1}{\sqrt{-g}} \text{ adds}
$$
\n
$$
e_1(\sqrt{g^{-1}\eta}) = 4 - \frac{1}{2}h^{\mu}_{\mu} + \frac{3}{8}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}(h^3),
$$
\n
$$
e_2 \text{ (11)} \quad \text{for } \mu \text{ is the constant,}
$$

$$
\frac{3}{4}
$$
 a mere constant to the action, and therefore it is totally irrelevant
unless one wants to have a dynamical metric $f_{\mu\nu}$ instead of η for

which it would contribute to its own cosmological constant. Terms
\nwith
$$
\beta_1, ..., \beta_{N-1}
$$
 make up the potential term for the graviton.
\nObviously, these polynomials can be described as coefficients in

the characteristic polynomial of M:
$$
e_4(\sqrt{g^{-1}\eta}) = 1 - \frac{1}{2}h^{\mu}_{\mu} + \frac{1}{8}(h^{\mu}_{\mu})^2 + \frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}(h^3).
$$
 (13)

$$
\sqrt{-g} = 1 + \frac{1}{2} h^{\mu}_{\mu} + \frac{1}{8} (h^{\mu}_{\mu})^2 - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(h^3).
$$
\n(14)

17 α easily given by multiplying $(10)-(12)$ by (14) :

$$
{}^{19} \quad \text{a shorthand notation which reads } [\mathcal{M}] = \mathcal{M}_{\mu}^{\mu}, [\mathcal{M}]^2 = (\mathcal{M}_{\mu}^{\mu})^2, \qquad \sqrt{-g} \cdot e_1(\sqrt{g^{-1}\eta}) = 4 + \frac{3}{2}h_{\mu}^{\mu} + \frac{1}{4}(h_{\mu}^{\mu})^2 - \frac{5}{8}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}(h^3), \qquad \text{as} \qquad \
$$

$$
\sqrt{-g} \cdot e_2(\sqrt{g^{-1}\eta}) = 6 + \frac{3}{2}h^{\mu}_{\mu} + \frac{1}{8}(h^{\mu}_{\mu})^2 - \frac{1}{2}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}(h^3),
$$
\n(16)

$$
6)
$$

$$
\frac{1}{27} = \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right).
$$
\n
$$
\sqrt{-g} \cdot e_3(\sqrt{g^{-1}\eta}) = 4 + \frac{1}{2} h^{\mu}_{\mu} - \frac{1}{8} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(h^3),
$$
\n
$$
\text{And one can prove a simple recurrent relation}
$$
\n
$$
= \frac{1}{2} \left([\mathcal{M}]^2 - [\mathcal{M}^2] \right).
$$

$$
\sqrt{-g} \cdot e_4(\sqrt{g^{-1}\eta}) = 1
$$
 exactly, and of course $\sqrt{-g} \cdot e_0 = \sqrt{-g}$ given
by (14).

 $\frac{32}{1}$ i=1 ³³ with $e_n = 0$ for $n > N$. ⁹⁸ order contribution to the action around Minkowski:

$$
\begin{array}{lll}\n\text{Set from (6)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 100 \\
\text{set from (6)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (6)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (7)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (8)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (9)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (9)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (9)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (9)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (9)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 101 \\
\text{set from (10)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt{-g} \cdot \beta_n e_n (\sqrt{g^{-1} \eta}) & & 102 \\
\text{set from (11)} & & V(h) \equiv m^2 \sum_{n=0}^{N} \sqrt
$$

which gives a condition

$$
\beta_0 = -3\beta_1 - 3\beta_2 - \beta_3.
$$

Being plugged back into the action, it yields the familiar result:

$$
V(h) - V(0) = \frac{m^2}{8} (\beta_1 + 2\beta_2 + \beta_3) \cdot \left(h^{\mu\nu} h_{\mu\nu} - (h^{\mu}_{\mu})^2 \right)
$$

$$
+\mathcal{O}(h^3).
$$

55 120 of bimetric theory *^gμν* ↔ *^fμν* , *βⁿ* ↔ *β^N*−*n*. It comes from the fact 57 $e_{N-n}(\mathcal{M})$ by dividing over det M. In particular, Note that we followed the usual path. However, these calcula-

be found explicitly assuming the trivial root of the unity matrix:
\n₆₀
$$
\sqrt{\mathbb{I}} = \mathbb{I}
$$
. Then the first terms of the Taylor expansion
\n
$$
\begin{aligned}\n &\text{for } \sqrt{\mathbb{I}} = \mathbb{I} - \frac{1}{2}H - \frac{1}{8}H^2 + \mathcal{O}(H^3) \\
 &\text{for } \sqrt{\mathbb{I}} = \mathbb{I} - \frac{1}{2}H - \frac{1}{8}H^2 + \mathcal{O}(H^3)\n \end{aligned}
$$
\n
$$
\begin{aligned}\n &\text{for } \sqrt{\mathbb{I}} = \mathbb{I} - \frac{1}{2}(\mathbb{I} - \frac{1}{8}H^2 + \mathcal{O}(H^3)) \\
 &\text{for } \sqrt{\mathbb{I} - H} = \mathbb{I} - \frac{1}{2}H - \frac{1}{8}H^2 + \mathcal{O}(H^3)\n \end{aligned}
$$
\n
$$
\begin{aligned}\n &\text{for } \sqrt{\mathbb{I} - H} = \mathbb{I} - \frac{1}{2}(\mathbb{I} - \frac{1}{8}H^2 + \mathcal{O}(H^3)) \\
 &\text{for } \sqrt{\mathbb{I} - H} = \mathbb{I} - \frac{1}{2}(\mathbb{I} - \frac{1}{8}H^2 + \mathcal{O}(H^3))\n \end{aligned}
$$

which also explains the mysterious disappearance of the $(h^{\mu}_{\mu})^2$ -

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