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On the gauge invariance of the decay rate of false vacuum

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ABSTRACT

We study the gauge invariance of the decay rate of the false vacuum for the model in which the scalar field responsible for the false vacuum decay has gauge quantum number. In order to calculate the decay rate, one should integrate out the field fluctuations around the classical path connecting the false and true vacua (i.e., so-called bounce). Concentrating on the case where the gauge symmetry is broken in the false vacuum, we show a systematic way to perform such an integration and present a manifestly gauge-invariant formula of the decay rate of the false vacuum.

There have been continuous interest in the theoretically correct calculation of the decay rate of the false vacuum. One of the recent motivations has been provided by the discovery of the Higgs boson at the LHC [1] and the precision measurement of the top quark mass at the LHC and Tevatron [2]; in the standard model, we are facing the possibility to live in a metastable electroweak vacuum with lifetime much longer than the age of the universe [3–9]. Furthermore, the false and true vacua may show up in various models of physics beyond the standard model. One important example is supersymmetric standard model in which the electroweak symmetry breaking vacuum may become unstable with the existence of the color or charge breaking vacuum at which colored or charged sfermion fields acquire vacuum expectation values; the condition that the electroweak vacuum has sufficiently large lifetime constrains the parameters in supersymmetric models [10–19]. Thus, detailed understanding of the decay of the false vacuum is important in particle physics and cosmology.

In [20–22], the calculation of the decay rate of the false vacuum was formulated with the so-called bounce configuration which is a solution of the 4-dimensional (4D) Euclidean equation of motion connecting false vacuum and true vacuum (more rigorously, the other side of the potential wall). The decay rate of the false vacuum per unit volume is given in the following form:

$$\gamma = \mathcal{A} e^{-\mathcal{B}}, \tag{1}$$

where $\mathcal{B}$ is the bounce action, while the prefactor $\mathcal{A}$ is obtained by integrating out field fluctuations around the bounce configuration as well as those around the false vacuum.

In gauge theories, if a scalar field with gauge quantum number acquires non-vanishing amplitude at the true or false vacuum, the gauge, Higgs and the ghost sectors contribute to $\mathcal{A}$. The decay rate should be calculated with the gauge-fixed Lagrangian which contains the gauge parameter ξ. In the present study, we concentrate on the gauge dependence (i.e., the ξ-dependence) of the decay rate of the false vacuum. Formally, the ξ-dependence of $\mathcal{A}$ should cancel out exactly. This is due to the fact that the decay rate is derived from the effective action of the bounce configuration, and also that the effective action for any solution of the equation of motion is assured to be gauge invariant [23,24]. In the actual calculation, however, the gauge independence is not manifest because the ξ-dependence should cancel out among the contributions of gauge field, Nambu–Goldstone (NG) boson, and Faddeev–Popov (FP) ghosts.[1] In particular, the gauge boson and the NG mode, whose fluctuation operator is ξ-dependent, mix with each other around the bounce configuration. This makes the study of the decay rate complicated. Furthermore, it is difficult to check

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[1] The gauge invariance of the effective potential of the model we consider was discussed in [25]; however, the scalar configuration was assumed to be space–time independent, and hence the result is not applicable to the present case. The gauge independence of the sphaleron transition rate was studied in [26] using functional determinant method which is also adopted in our analysis.

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the gauge independence even numerically because a stable numerical implementation proposed so far requires $\xi = 1$.

In this letter, we show a procedure to integrate out the field fluctuations, which gives rise to a manifestly gauge invariant expression of the decay rate overcoming the difficulties mentioned above. In the current study, we concentrate on the case where

1. the gauge symmetry is $U(1)$,[2]
2. there is only one charged scalar field Φ which affects the decay of the false vacuum,
3. the $U(1)$ symmetry is spontaneously broken in the false vacuum.

More general cases, in particular, the case where the $U(1)$ symmetry is preserved at the false vacuum, is discussed in [27].

First, let us explain the set up of our analysis. The Euclidean Lagrangian is given by

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + [(\partial_\mu + igA_\mu)\Phi^\dagger][(\partial_\mu - igA_\mu)\Phi] + V + \mathcal{L}_{\text{G.F.}} + \mathcal{L}_{\text{ghost}}, \tag{2}$$

where A_μ is the gauge field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and V is the scalar potential. In addition, $\mathcal{L}_{\text{G.F.}}$ and $\mathcal{L}_{\text{ghost}}$ are the gauge-fixing term and the terms containing FP ghosts (denoted as c and $\bar{c}$), respectively. We use the following gauge-fixing function[3]:

$$\mathcal{F} = \partial_\mu A_\mu - 2\xi g(\text{Re}\Phi)(\text{Im}\Phi) = \partial_\mu A_\mu + \frac{i}{2}\xi g(\Phi^2 - \Phi^{\dagger 2}), \tag{3}$$

with which

$$\mathcal{L}_{\text{G.F.}} = \frac{1}{2\xi}\mathcal{F}^2, \tag{4}$$

and

$$\mathcal{L}_{\text{ghost}} = \bar{c}\left[-\partial_\mu\partial_\mu + \xi g^2(\Phi^2 + \Phi^{\dagger 2})\right]c. \tag{5}$$

The scalar potential V has true and false vacua. We assume that the true and false vacua exist at the tree-level; we do not consider the case where the second vacuum is radiatively generated. The field configuration of the false vacuum is expressed as[4]

$$(A_\mu, \Phi)_{\text{false vacuum}} = (0, v/\sqrt{2}), \tag{6}$$

with v being a constant which is non-vanishing in this letter.

The false vacuum decay is dominated by the classical path, so-called the bounce [20]. When $v \neq 0$, the bounce solution, which is $O(4)$ symmetric [28,29], is given in the following form:

$$(A_\mu, \Phi)_{\text{bounce}} = (0, \bar{\phi}(r)/\sqrt{2}), \tag{7}$$

where $r \equiv \sqrt{x_\mu x_\mu}$ is the radius of the 4D Euclidean space. Here, the function $\bar{\phi}$ is a solution of the classical equation of motion:

$$\left[\partial_r^2\Phi + \frac{3}{r}\partial_r\Phi - V_\Phi\right]_{\Phi\to\bar{\phi}/\sqrt{2}} = 0, \tag{8}$$

where V_Φ denotes the derivative of the scalar potential with respect to Φ. It also satisfies the following boundary conditions:

$$\partial_r\bar{\phi}(r=0) = 0, \tag{9}$$

$$\bar{\phi}(r=\infty) = v. \tag{10}$$

We assume that $\bar{\phi}$ is a real function of r. At $r \to \infty$, $\bar{\phi}$ settles on the false-vacuum; in such a limit, $\bar{\phi}$ (approximately) obeys the following equation:

$$\partial_r^2\bar{\phi} + \frac{3}{r}\partial_r\bar{\phi} - m_h^2(\bar{\phi} - v) \simeq 0, \tag{11}$$

where m_h is the mass of the (massive) scalar boson around the false vacuum. Then, the asymptotic behavior of $\bar{\phi}$ can be expressed as

$$\bar{\phi}(r\to\infty) \simeq v + \kappa\frac{e^{-m_h r}}{r^{3/2}}, \tag{12}$$

with κ being a constant.

For the calculation of the decay rate of the false vacuum, it is necessary to integrate out the fluctuations around the bounce. The gauge and scalar fields are decomposed around the bounce as

$$A_\mu = a_\mu, \quad \Phi = \frac{1}{\sqrt{2}}\left(\bar{\phi} + h + i\varphi\right), \tag{13}$$

where the "Higgs" mode h and the "NG" mode φ are real fields. We expand the field fluctuations as[5]

$$\begin{aligned} a_\mu(x) \ni\ & \alpha_S(r)\frac{x_\mu}{r}\mathcal{Y}_{J,m_A,m_B} + \alpha_L(r)\frac{r}{L}\partial_\mu\mathcal{Y}_{J,m_A,m_B} \\ & + \alpha_{T1}(r)i\epsilon_{\mu\nu\rho\sigma}V_\nu^{(1)}L_{\rho\sigma}\mathcal{Y}_{J,m_A,m_B} \\ & + \alpha_{T2}(r)i\epsilon_{\mu\nu\rho\sigma}V_\nu^{(2)}L_{\rho\sigma}\mathcal{Y}_{J,m_A,m_B}, \end{aligned} \tag{14}$$

$$h(x) \ni \alpha_h(r)\mathcal{Y}_{J,m_A,m_B}, \tag{15}$$

$$\varphi(x) \ni \alpha_\varphi(r)\mathcal{Y}_{J,m_A,m_B}, \tag{16}$$

where $\mathcal{Y}_{J,m_A,m_B}$ denotes the 4D hyperspherical harmonics; the eigenvalues of S_A^2, S_B^2, $S_{A,3}$ $S_{B,3}$ (with S_A and S_B being generators of the rotational group of the 4D Euclidean space, i.e., $SU(2)_A \times SU(2)_B$) are $J(J+1)$, $J(J+1)$, m_A, and m_B, respectively. Notice that $J = 0, \frac{1}{2}, 1, \cdots$. In addition, $V_\nu^{(1)}$ and $V_\nu^{(2)}$ are (arbitrary) two independent vectors, $L_{\rho\sigma} \equiv \frac{i}{\sqrt{2}}(x_\rho\partial_\sigma - x_\sigma\partial_\rho)$, and

$$L \equiv \sqrt{4J(J+1)}. \tag{17}$$

For $J > 0$, the fluctuation operator for $(\alpha_S, \alpha_L, \alpha_\varphi)$ is obtained as

$$\mathcal{M}_J^{(S,L,\varphi)} \equiv \begin{pmatrix} -\Delta_J + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & 2g\bar{\phi}' \\ -\frac{2L}{r^2} & -\Delta_J - \frac{1}{r^2} + g^2\bar{\phi}^2 & 0 \\ 2g\bar{\phi}' & 0 & -\Delta_J + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix} + \left(1 - \frac{1}{\xi}\right)\begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{18}$$

[2] The application of our prescription to the case of non-abelian gauge symmetry is straightforward.

[3] Previous studies used different type of the gauge-fixing functions: $\partial_\mu A_\mu - \sqrt{2}\xi g\bar{\phi}\text{Re}\Phi$, around the bounce (i.e., $\Phi = \bar{\phi}/\sqrt{2}$), and $\partial_\mu A_\mu - \sqrt{2}\xi gv\text{Im}\Phi$, around the false vacuum (i.e., $\Phi = v/\sqrt{2}$). Expanding the fields around the solution of the classical equation of motion, we obtain the same gauge-fixing functions as the previous studies at least at the one-loop level, although our gauge-fixing function can be used both around the bounce and around the false vacuum.

[4] The field amplitude at the false vacuum (as well as the bounce configuration) may be shifted due to loop effects; the shifts are ξ-dependent in general. However, at the one-loop level, the shifts do not affect the extremum values of the effective action to which the decay rate of the false vacuum is related.

[5] For notational simplicity, we omit the subscripts J, m_A, and m_B from the radial function α's, and the summations over J, m_A, and m_B are implicit.