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Complementarity and stability conditions

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ABSTRACT

We discuss the issue of complementarity between the confining phase and the Higgs phase for gauge theories in which there are no light particles below the scale of confinement or spontaneous symmetry breaking. We show with a number of examples that even though the low energy effective theories are the same (and trivial), discontinuous changes in the structure of heavy stable particles can signal a phase transition and thus we can sometimes argue that two phases which have different structures of heavy particles that cannot be continuously connected and thus the phases cannot be complementary. We discuss what this means and suggest that such "stability conditions" can be a useful physical check for complementarity.

1. Introduction

This note is an attempt to understand better the classic papers by Fradkin and Shenker [1], Banks and Rabinovici [2], 't Hooft [3] and Dimopoulos, Raby and Susskind [4,5] related to complementarity between the Higgs and confining phases in gauge theories.[1] In model building, this is important because it sometimes happens that one takes a Higgsed theory that is perturbatively calculable for small couplings and pushes it into regions in which perturbation theory is questionable. If the Higgs phase and confining phase are complementary, that is if there is no phase transition separating the Higgs phase and confining phase, then one may hope that this will give a picture of the physics that is qualitatively correct even if it is not quantitatively reliable. But if the two phases are genuinely different, then you have no right to expect that this procedure will make any sense at all.

A recent example is an $SU(N+3) \times SU(3) \times U(1)$ model that was suggested as a possible explanation of the di-photon excess at 750 GeV [8]. The model has $(N+3, \overline{3})$ scalar field ξ that is trying to break the symmetry down to $SU(N) \times SU(3) \times U(1)$.[2] In the limit in which only one of the couplings gets strong, we can think of the strong non-Abelian group as the gauge symmetry and treat the other approximately as a global symmetry.

If $SU(3)$ gets strong and $SU(N+3)$ is global, the issue is easy. Here, I think that there is no hope of complementarity. Because in this case, in the Higgs phase, we have the $SU(N+3) \times U(1)$ global symmetry broken down to $SU(N) \times SU(3) \times U(1)$. There is a coset space

$$\frac{SU(N+3)}{SU(N) \times SU(3)} \tag{1.1}$$

describing an $(N, \overline{3})$ of massless Goldstone bosons in the Higgs phase and there is no unbroken gauge symmetry And even if the $SU(N+3)$ is weakly gauged, the heavy vectors are light and still present in the low energy theory.

In the confining $SU(3)$ theory, there is no reason for the global $SU(N+3)$ to break and no reason for anything to be light. So in this situation, the phases are distinguished by different symmetries and different massless particles in the low energy theory.

What happens if $SU(N+3)$ gets strong? Then presumably the $SU(3)$ is unbroken both in the confining phase and in the Higgs phase. So this could perhaps be complementary. In the Higgs phase we have massless $SU(N)$ gauge bosons, and the rest of the $SU(N+3)$ gauge bosons have mass of order gv. And Λ_N is of the same order of magnitude times the exponential factor that goes to 1 as the coupling gets large. Thus in the gauge invariant spectrum there are glueballs and bound states of heavy vectors. As the coupling increases, all of these things get heavy! Likewise, in the confining phase of the full $SU(N+3)$ theory, we expect that all the particle states will have mass of the order of the $SU(N+3)$ confinement scale or greater.

Thus in both the confining phase and the Higgs phase, the low energy theories are trivial. This is consistent with complementarity, and in this case, we believe that the phases are in fact complementary. However, in general, the equivalence of the effective low energy theories in the confining and Higgs phases [3] is not a suf-

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[1] See also [6]. One other reference that might be useful is [7].

[2] There are no other matter fields that carry the $SU(N+3)$.

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ficient condition for complementarity.[3] And we suggest another diagnostic for complementarity that can be useful.

It may be that even when the low energy particles and symmetries acting on them are identical, there are sectors describing heavy particles in the two phases with different properties that distinguish the two phases. The property that we will focus on is stability. In a sense, a heavy stable particle is part of the effective low energy theory because if something puts one in the low-energy world, it stays there and its interactions do not involve any high-energies. [10] Stability conditions can be an easy and very physical way of identifying this situation.

It is important to note that stability for a particular set of parameters is not enough because complementarity is about how the physics changes as parameters change. We are interested in the situation in which stability is guaranteed independent of the phase space. An example of this is a theory with a conserved quantized charge. A conserved charge divides the space of physical states up into sectors with definite charge, separated by superselection rules. In a theory with a single conserved charge, the sector with the lowest non-zero positive charge must contain stable states — either a single particle with the minimum charge or a collection of stable particles with total charge equal to the minimum. There is stability here, but it is not a property of the particle. We can certainly imagine changing the parameters in the theory continuously to make some a different particle carrying the conserved charge (not necessarily the same value of the charge) the lightest particle. And indeed, no single particle with the lowest charge has to exist at all. But at least some particles carrying the charge will always be stable so long as the charge is conserved. We might say that each sector of charged states is unconditionally stable, because there is always some combination of particles that is the lightest state with the appropriate charge.

As a very explicit (and fairly silly) example imagine a world with a conserved charge and three types of charged particle, A, B and C with charges 2, 3, and 5 respectively. The lowest positive charge is 1, and the stable states in the charge 1 sector could be $\bar{A}B$, $\bar{A}\bar{A}C$ or $\bar{C}BB$, depending on the particle masses. Charge conservation guarantees that two of the particle types are stable, and which two are actually stable depends on the masses, but the charge 1 sector is stable independent of the details of the masses..

If in a phase transition, the lowest positive charge changes, then even if the light particles in the two phases are qualitatively similar, the possible structures of stable particles in the effective low energy theory must be different in the two phases. There is then no way to get continuously from one effective theory to the other, and the two phases cannot be complementary.

In the remainder of this note, we will give a series of examples based on familiar $SU(N)$ groups. We hope they will convince the reader that this is an interesting approach.

2. SU(5) with a scalar 10

As a warm-up, and to get the reader used to the style of analysis, consider an $SU(5)$ theory with a single 10 of scalars, $\xi^{jk} = -\xi^{kj}$. The most general renormalizable Lagrangian has a global $U(1)$ symmetry, and for a range of parameters, ξ develops a VEV that can be put in the form[4]

$$\langle \xi \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -v \\ 0 & 0 & 0 & v & 0 \end{pmatrix} \tag{2.2}$$

This breaks the $SU(5)$ gauge symmetry down to $SU(3) \times SU(2)$, under which ξ transforms as

$$(\overline{3}, 1) + (3, 2) + (1, 1) \tag{2.3}$$

with the VEV in the $(1, 1)$. The $(3, 2)$ and the imaginary part of the $(1, 1)$ are eaten by the Higgs mechanism producing a $(3, 2)$ and $(1, 1)$ of massive vector bosons.

There is also a global $U(1)$ symmetry that is a combination of the original global $U(1)$ and the $U(1)$ generator of the $SU(5)$ that commutes with $SU(3) \times SU(2)$. The $(1, 1)$ in (2.3) must be neutral under the unbroken symmetry, so the charges must look like

$$(\overline{3}, 1)_2 + (3, 2)_1 + (1, 1)_0 \tag{2.4}$$

in some arbitrary normalization, and because the $U(1)$ charge of the multiplet must be the average charge of the multiplet after symmetry breaking, we know that ξ is a $10_{6/5}$. The condensate also breaks the global 5-ality of the $SU(5)$ theory. down to triality×duality for the $SU(3) \times SU(2)$ In the Higgsed theory, the uneaten $(\overline{3}, 1)$ of scalars has triality 2 and charge 2, the $(3, 2)$ massive gauge boson has triality 1, duality 1 and charge 1.

In both the Higgs phase and the confining phase, heavy particles carry a quantized conserved charge. Now we can examine the stable sectors in the Higgs phase and the confining phase. In this case, they match up perfectly. In the Higgs phase, all the triality and duality zero gauge singlet combinations like 3 $(\overline{3}, 1)_2$ scalars or 6 $(3, 2)_1$ massive vector bosons all have $U(1)$ charges which are multiples of 6. In the confining theory the 5-ality zero states are combinations of 5 $10_{6/5}$ scalars, which have the same property. The lowest positive charge is 6 in both cases.

Thus the stability conditions do not distinguish between this Higgs phase and the confining phase, and this is consistent with complementarity.

3. SU(5) with a scalar 15

Contrast the model discussed in section 2 with an $SU(5)$ theory with a single 15 of scalars, $\xi^{jk} = \xi^{kj}$. The most general renormalizable Lagrangian again has a global $U(1)$ symmetry, and for a range of parameters, ξ develops a VEV that can be put in the form[5]

$$\langle \xi \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & v \end{pmatrix} \tag{3.5}$$

This breaks the $SU(5)$ gauge symmetry down to an $SU(4)$, under which ξ transforms as

$$10 + 4 + 1 \tag{3.6}$$

with the VEV in the 1. The 4 and the imaginary part of the 1 are eaten by the Higgs mechanism producing a 4 and 1 of massive vector bosons.

There is also a global $U(1)$ symmetry that is a combination of the original global $U(1)$ and the $U(1)$ generator of the $SU(5)$ that

[3] This has been emphasized in a very different context in [9].

[4] See section A.2. Note that this statement is not trivial, and such details are too often ignored in treatments of Higgs theories. However, here, we want to focus on other things, so in this and subsequent sections, we will relegate the discussion of the potentials to Appendix A.

[5] See section A.3.

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