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# Quantum non-equilibrium effects in rigidly-rotating thermal states

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ABSTRACT

Based on known analytic results, the thermal expectation value of the stress-energy tensor (SET) operator for the massless Dirac field is analysed from a hydrodynamic perspective. Key to this analysis is the Landau decomposition of the SET, with the aid of which we find terms which are not present in the ideal SET predicted by kinetic theory. Moreover, the quantum corrections become dominant in the vicinity of the speed of light surface (SOL). While rigidly-rotating thermal states cannot be constructed for the Klein–Gordon field, we perform a similar analysis at the level of quantum corrections previously reported in the literature and we show that the Landau frame is well-defined only when the system is enclosed inside a boundary located inside or on the SOL. We discuss the relevance of these results for accretion disks around rapidly-rotating pulsars.



## 1. Introduction

In relativistic fluid dynamics, global thermal equilibrium can be attained if the product $\beta u^\mu$ between the inverse local temperature $\beta$ and the four-velocity $u^\mu$ of the flow satisfies the Killing equation [1–5]. A special property of thermal equilibrium is that the stress-energy tensor (SET) $T^{\mu\nu}_{\rm eq} = (E+P)u^\mu u^\nu + P g^{\mu\nu}$ corresponds to that of an ideal fluid of energy density $E$ and pressure $P$ [2,6–8].[1] In this letter, we will show that a quantum field theory (QFT) computation of the SET for rigidly-rotating thermal states (RRTS) contains non-ideal terms, as well as corrections to $E$ which become important near the speed of light surface (SOL). We discuss the relevance of these corrections in the context of an astrophysical application.

## 2. Kinetic theory analysis

In space–times with axial symmetry, RRTS in thermal equilibrium can be described using the Killing vector corresponding to rotations about the $z$ axis, i.e., $\beta u = \beta_0(\partial_t + \Omega\partial_\varphi)$, where $\Omega$ is the angular velocity of the rotating state [7]. On Minkowski space, the particle four-flow $N^\mu_{\rm eq}$ and stress-energy tensor $T^{\mu\nu}_{\rm eq}$ corresponding to RRTS are given by:

$$N^\mu_{\rm eq} = n u^\mu, \qquad T^{\mu\nu}_{\rm eq} = (E+P)u^\mu u^\nu + P g^{\mu\nu}, \tag{1}$$

while $\beta$ and $u = u^\mu \partial_\mu$ are given by:

$$\beta = \gamma^{-1}\beta_0, \qquad u = \gamma(\partial_t + \Omega\partial_\varphi), \tag{2}$$

where $\gamma$ is the Lorentz factor of a co-rotating observer at distance $\rho$ from the $z$ axis:

$$\gamma = \left(1 - \rho^2\Omega^2\right)^{-1/2}. \tag{3}$$

The Killing vector $\beta u$ becomes null on the SOL, where $\rho\Omega \to 1$ and co-rotating observers travel at the speed of light. From Eq. (2), it can be seen that the temperature $\beta^{-1}$ diverges as the SOL is approached. The energy density $E$ for massless particles obeying Fermi–Dirac (F–D) and Bose–Einstein (B–E) statistics is given by [6]:

$$E_{\rm F-D} = \frac{7\pi^2\gamma^4}{60\beta_0^4}, \qquad E_{\rm B-E} = \frac{\pi^2\gamma^4}{30\beta_0^4}, \tag{4}$$

while $P = E/3$. Since $E$ and $P$ diverge as inverse powers of the distance to the SOL, RRTS are well-defined only up to the SOL. While such divergent states clearly cannot occur in nature, rigid rotation can be induced in astrophysical systems, such as accretion disks around rapidly-rotating neutron stars or magnetars, where the intense magnetic field can lock charged particles into rigid rotation.[2]

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[1] We use Planck units with $c = \hbar = k_B = 1$, while the metric signature is $(-,+,+,+)$.

[2] In such systems, various mechanisms prevent the violation of special relativity [9].

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We investigate the role of quantum corrections in such systems in Sec. 6.

## 3. Stress-energy tensor decompositions

Before discussing the quantum analogue of Eqs. (4), the tools necessary to analyse the SET in out of equilibrium states must be introduced. The main difficulty comes due to the equivalence between mass and energy in special relativity, which makes the distinction between the velocity $u^\mu$ and the heat flux $q^\mu$ ambiguous. For a general (time-like) choice of $u^\mu$, $N^\mu$ can be decomposed as [10]:

$$N^\mu = nu^\mu + V^\mu, \tag{5}$$

where $n = -u_\mu N^\mu$ and the flow of particles in the local rest frame (LRF) $V^\mu$ is given by:

$$V^\mu = \Delta^\mu{}_\nu N^\nu \tag{6}$$

In the above, $\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$ is the projector on the hypersurface orthogonal to $u^\mu$. The decomposition of the SET reads:

$$T^{\mu\nu} = Eu^\mu u^\nu + (P + \varpi)\Delta^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}, \tag{7}$$

where the dynamic pressure $\varpi$, flow of energy in the LRF $W^\mu$ and shear stress $\pi^{\mu\nu}$, together with $V^\mu$, represent non-equilibrium terms. The quantities on the right hand side of Eq. (7) can be obtained through:

$$E = u^\mu u^\nu T_{\mu\nu}, \quad P + \varpi = \frac{1}{3}\Delta^{\mu\nu} T_{\mu\nu}, \quad W^\mu = -\Delta^{\mu\nu} u^\lambda T_{\nu\lambda},$$

$$\pi^{\mu\nu} = \left(\Delta^{\mu\lambda}\Delta^{\nu\sigma} - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\lambda\sigma}\right) T_{\lambda\sigma}, \tag{8}$$

For a massless fluid, $\varpi = 0$. The heat flux $q^\mu$ is defined as [10]:

$$q^\mu = W^\mu - \frac{E + P}{n} V^\mu. \tag{9}$$

In the Eckart (particle) frame [2,8,11], $u_e^\mu$ is defined as the unit vector parallel to $N^\mu$. Observers in the LRF of the Eckart frame see a flow of energy ($W_e^\mu = q_e^\mu$) and no flow of particles ($V_e^\mu = 0$). Since $N^\mu$ cannot be obtained using the QFT approach considered in this paper, the Eckart velocity $u_e^\mu$ cannot be defined. Hence, we will not consider the Eckart frame further in this paper.

In the Landau (energy) frame [2,8,12], $u^\mu \equiv u_L^\mu$ is defined as the eigenvector of $T^\mu{}_\nu$ corresponding to the (real, positive) Landau energy density $E_L$:

$$T^\mu{}_\nu u_L^\nu = -E_L u_L^\mu, \tag{10}$$

such that $W_L^\mu = 0$, which implies that there is no energy flux in the LRF. Since $V_L^\mu = -\frac{n_L}{E_L + P_L} q_L^\mu$ is in general non-zero, an observer in the LRF of the Landau frame will detect a non-vanishing particle flux.

Finally, the $\beta$-frame (thermometer frame) for the case of rigid rotation is defined with respect to [4]:

$$u_\beta = \gamma(\partial_t + \Omega\partial_\varphi). \tag{11}$$

A special property of the $\beta$-frame is that the LRF temperature is highest compared to the temperature measured with respect to any other frame [4]. In general, $V_\beta^\mu$ and $W_\beta^\mu$ do not vanish, such that the $\beta$-frame is a mixed particle-energy frame [13]. Due to the simplicity of its construction, we will start the analysis of the quantum SET with respect to the $\beta$-frame.

## 4. Klein–Gordon field

We now analyse the construction of RRTS from a QFT perspective. A first surprise comes from the analysis of the RRTS of the Klein–Gordon field: in the unbounded Minkowski space, there exist modes which have a non-vanishing Minkowski energy $\omega$ (i.e., with respect to the static Hamiltonian $H_s = i\partial_t$), while their co-rotating energy $\widetilde{\omega} = \omega - \Omega m$, measured with respect to the rotating Hamiltonian $H_r = i(\partial_t + \Omega\partial_\varphi)$, vanishes. For such modes, the Bose–Einstein density of states factor $(e^{\beta\widetilde{\omega}} - 1)^{-1}$ diverges, yielding divergent thermal expectation values (t.e.v.s) at every point in the space–time [14,15]. The kinetic theory result (4) is clearly unaffected by this vanishing co-rotating energy modes catastrophy. Indeed, the problematic modes are no longer present in the QFT formulation if the system is enclosed within a boundary placed inside or on the SOL [15,16]. Furthermore, a recent perturbative QFT analysis allows the computation of quantum corrections to the kinetic theory SET [17], which we will analyse in detail in what follows. For completeness, we present an analysis of the connection between these perturbative results and the non-perturbative QFT approach in Appendix A.

Substituting the results in Table III of Ref. [17] into Eq. (34) in Ref. [17] yields the following $\beta$-frame (2) decomposition of the SET:

$$E_\beta = \frac{\pi^2\gamma^4}{30\beta_0^4} + \frac{\Omega^2\gamma^6}{36\beta_0^2}, \qquad W_\beta = \frac{\Omega^3\gamma^7}{18\beta_0^2}\left(\rho^2\Omega\partial_t + \partial_\varphi\right),$$

$$\pi_\beta^{\mu\nu} = \frac{\Omega^2\gamma^6}{54\beta_0^2}\begin{pmatrix} \gamma^2 - 1 & 0 & \Omega\gamma^2 & 0 \\ 0 & 1 & 0 & 0 \\ \Omega\gamma^2 & 0 & \rho^{-2}\gamma^2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}, \tag{12}$$

where $\omega = \Omega\gamma^2\partial_z$, $a = \rho\Omega^2\gamma^2\partial_\rho$ and $\gamma = \beta_0^2\Omega^3\gamma^3(\rho^2\Omega\partial_t + \partial_\varphi)$ were used in Eq. (34) of Ref. [17]. Compared to the kinetic theory result (1), the quantum SET contains non-vanishing contributions in the form of the non-ideal terms $W^\mu$ and $\pi^{\mu\nu}$. Moreover, the second term in $E_\beta$ (12) represents a correction to $E_{\text{B-E}}$ (4) which becomes dominant in the vicinity of the SOL due to the $\gamma^6$ factor.

The construction of the Landau frame yields:

$$E_L = \frac{E_\beta}{3} - \frac{1}{2}\hat{\boldsymbol{W}}_\beta \cdot \boldsymbol{\pi}_\beta \cdot \hat{\boldsymbol{W}}_\beta + \sqrt{\left(\frac{2E_\beta}{3} + \frac{1}{2}\hat{\boldsymbol{W}}_\beta \cdot \boldsymbol{\pi}_\beta \cdot \hat{\boldsymbol{W}}_\beta\right)^2 - W_\beta^2}, \tag{13}$$

$$u_L^\mu = \sqrt{\frac{E_L + \frac{1}{3}E_\beta + \hat{\boldsymbol{W}}_\beta \cdot \boldsymbol{\pi}_\beta \cdot \hat{\boldsymbol{W}}_\beta}{2(E_L - \frac{1}{3}E_\beta + \frac{1}{2}\hat{\boldsymbol{W}}_\beta \cdot \boldsymbol{\pi}_\beta \cdot \hat{\boldsymbol{W}}_\beta)}} \times \left(u_\beta^\mu + \frac{W_\beta^\mu}{E_L + \frac{1}{3}E_\beta + \hat{\boldsymbol{W}}_\beta \cdot \boldsymbol{\pi}_\beta \cdot \hat{\boldsymbol{W}}_\beta}\right), \tag{14}$$

where $W_\beta^2 = \rho^2\Omega^6\gamma^{12}/324\beta_0^4 \ge 0$, $\hat{\boldsymbol{W}}_\beta \equiv W_\beta/\sqrt{W_\beta^2}$ and $\hat{\boldsymbol{W}}_\beta \cdot \boldsymbol{\pi}_\beta \cdot \hat{\boldsymbol{W}}_\beta = \Omega^2\gamma^6/54\beta_0^2$. Surprisingly, the Landau frame is well-defined only for $0 \le \rho\Omega \le (\rho\Omega)_{\text{lim}}$, where

$$(\rho\Omega)_{\text{lim}} = \sqrt{x^2 + x + 1} - x, \qquad x = \frac{5}{4\pi^2}(\beta_0\Omega)^2. \tag{15}$$

When $\rho\Omega > (\rho\Omega)_{\text{lim}}$, $E_L$ is no longer real. It can be seen that $(\rho\Omega)_{\text{lim}}$ decreases from 1 at $\beta_0\Omega = 0$ (large temperatures or slow rotation) down to 0.5 as $\beta_0\Omega \to \infty$.

We are again forced to regard the RRTS of the Klein–Gordon field as ill-defined. The natural question to ask is whether the problem with defining the Landau frame persists when the system