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Holographic corrections to the Veneziano amplitude

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ABSTRACT

We propose a holographic computation of the $2 \to 2$ meson scattering in a curved string background, dual to a QCD-like theory. We recover the Veneziano amplitude and compute a perturbative correction due to the background curvature. The result implies a small deviation from a linear trajectory, which is a requirement of the UV regime of QCD.

1. Introduction

The Veneziano amplitude, aimed at describing meson scattering, marks the birth of string theory [1]. While the Veneziano amplitude exhibits several attractive properties, such as the duality between the s-channel and the t-channel, and a phenomenological success in the Regge regime, it also suffers from an undesired exponential behaviour in the high energy regime.

The properties of the Veneziano amplitude are closely related to the phenomenon of confinement. Makeenko and Olesen [2] showed that the amplitude can be reproduced from large-N QCD, by representing the amplitude as a sum over Wilson loops. A crucial unrealistic assumption, is that all Wilson loops (even small loops) admit an area law.

In a recent attempt to derive the Veneziano amplitude from large-N QCD [3], the sum over Wilson loops of [2] was written by using holography [4] as a sum over string worldsheets. The sum includes all Wilson loops that pass via the positions of the mesons. The Veneziano amplitude was obtained under the assumption that Wilson loops are saturated by flat space configurations that sit on the IR cut-off (hence leading to an area law). The flat space approximation can be achieved by bringing the IR cut-off close to the UV cut-off (the space boundary).

Thus one can attribute the failure of the Veneziano amplitude in the high energy regime to the flat space approximation, which is identical to the assumption that all Wilson loops admit an area law. It is therefore desired to accommodate small Wilson loops. In the dual string theory it means that one has to calculate string amplitudes in curved space. Even if that could be done, an exact dual of large-N QCD is not known, hence it is not clear which non-linear sigma model should be used.

In this paper we propose a method to improve the scattering amplitude, by incorporating curvature effects from the dual geometry. In particular we study the effect of worldsheet fluctuations in the vicinity of the IR cut-off. As the worldsheet fluctuates it probes part of the UV geometry. To this end we include in the string sigma model an interaction term between the flat 4d coordinates and extra dimensions and calculate a two-loop perturbative correction to the propagator. While we carry out the calculation by using Witten's model of the dual of Yang–Mills theory [5], we believe that the sigma model we use characterizes generic confining holographic models. The result of the calculation is a correction to the Veneziano amplitude in the form of a deviation from a linear Regge trajectory $\alpha(s) \sim s^{(1-\rho^3 \log^2 s)}$, with $\rho^3 \log^2 s \ll 1$.

Our approach is distinct from earlier studies of Pomeron scattering using the AdS/CFT correspondence [6]. The novelty, apart from considering meson scattering (open string amplitude), is that we include a perturbative correction to the X^μ propagator.

Our conclusion about the non-linearity of the Regge trajectory at small angular momenta is similar to that of previous studies [7, 8]. It is consistent with empirical data [8].

2. Setting up the duality

The starting point of this framework is the aforementioned identity involving a sum over all sizes of Wilson loops. To compute this four-point function we make use of the Worldline formalism, namely the following equality for the fermion determinant of $SU(N_c)$ QCD with N_f flavours, in terms of expectation values of super-Wilson loops of length T:

$$\mathcal{Z} = \int DA \exp(-S_{\text{YM}}) \left(\det\left(i\not{D}\right)\right)^{N_f} ,$$

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$$\left(\det\left(i\not{D}\right)\right)^{N_f} = \exp\left(-\frac{N_f}{2}\mathrm{Tr}\int_0^\infty \frac{dT}{T}\,\langle \mathcal{W}_T[A]\rangle\right), \tag{1}$$

$$\langle \mathcal{W}_T[A]\rangle = \int DxD\psi\, e^{-\frac{1}{2}\int_0^T d\tau\,(\dot{x}^2+\psi\cdot\dot{\psi})} e^{i\int_0^T d\tau\,(\dot{x}\cdot A - \frac{1}{2}\psi\cdot F\cdot\psi)}.$$

This exponential can then be expanded in powers of N_f/N_c, so that in the 't Hooft large N_c limit only the linear term in Wilson loops needs to be considered. We will use this approximation (the so-called "quenched approximation") for the following computation of a generic meson 4-point function [3]:

$$\left\langle \prod_{i=1}^4 q\bar{q}(x_i)\right\rangle = \frac{1}{\mathcal{Z}}\prod_{i=1}^4 \frac{\delta}{\delta J(x_i)}\int DA\exp(-S_{\mathrm{YM}}) \times \left(-\frac{N_f}{2}\mathrm{Tr}\int_0^\infty \frac{dT}{T}\,\langle \mathcal{W}_T[A,J]\rangle\right)|_{J=0} \tag{2}$$

with $\langle \mathcal{W}_T[A,J]\rangle$ the worldline action of a theory coupled to a meson source $S_J = \int d^4x\, J\bar{q}q$ [9]. Taking functional derivatives in terms of J creates delta functions constraining the various loops at hand to pass through the points where the operators are inserted. We therefore arrive at the following formal expression for the scattering amplitude

$$\left\langle \prod_{i=1}^4 q\bar{q}(x_i)\right\rangle = \frac{1}{\mathcal{Z}}\int DA\exp(-S_{\mathrm{YM}}) \times \left(-\frac{N_f}{2}\mathrm{Tr}\int_0^\infty \frac{dT}{T}\,\langle \mathcal{W}_T[A]\rangle\,|_{x_1,x_2,x_3,x_4}\right), \tag{3}$$

where the sum over all Wilson loops has been imposed to include only those loops passing through the 4 points where the meson operators have been inserted.

The gauge/gravity duality implies that, for the field theory on the boundary, a generic Wilson loop's expectation value is related to the expectation value of a string worldsheet hanging from the loop on the boundary down into the bulk. More precisely, a single Wilson loop expectation value is obtained as the saddle of a sum over string worldsheets, as explained in section 3 of [11]. It was then proposed that the contribution of "quenched flavours" to the gauge theory partition function is dual to a sum over all string worldsheet with a topology of a disk that terminate on the boundary of the AdS space [10]

$$\int DA\exp(-S_{\mathrm{YM}}) \times \left(-\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{dT}{T}\,\langle \mathcal{W}_T[A]\rangle\right) = \int [DX][Dg]\exp\left(-\frac{1}{2\pi\alpha'}\int d^2\sigma\, G_{MN}\partial_\alpha X^M\partial_\beta X^N g^{\alpha\beta}\right). \tag{4}$$

This identification is possible because we have fixed a flat worldsheet gauge $g^{\alpha\beta} = \delta^{\alpha\beta}$, we are not interested in higher genus contributions. Then, schematically, the above equality holds because both path integration measures on either side sum up area-behaved integrands over all possible shapes and sizes of their boundaries. We are therefore able to write the following expression for the amplitude [10]

$$\mathcal{A}(k_{i=1\ldots4}) = \oint \prod_{i=1}^4 d\sigma_i \int [DX]We^{ik_i^\mu X_\mu(\sigma_i)} \times \exp\left(-\frac{1}{2\pi\alpha'}\int d^2\sigma\, G_{MN}\partial_\alpha X^M\partial^\alpha X^N\right), \tag{5}$$

where $We^{ik_i^\mu X_\mu(\sigma_i)}$ are meson vertex operators in the dual string theory, inserted at different points on the worldsheet, each associated to a momentum k_i. W encodes details of spin ($W=1$ for the tachyon and $W = \epsilon\partial_\sigma X$ for a vector for the bosonic string), which only really affects the intercept α_0 of the Regge trajectory, $\alpha(s) = \alpha_0 + \alpha' s$. As we shall see the calculated perturbative correction is not valid near $s=0$. In addition, there may be additional subleading phenomena contributing to a shift of the intercept, hence we can make no statements thereupon. The expectation values of the vertex operators are taken with respect to the Polyakov-type non-linear sigma model action over the string worldsheet into a curved space. Choosing the correct background is crucial in this matter. Since we have done away with flavour degrees of freedom via the worldline formalism, we only need to pick a space dual to pure Yang–Mills. For our purposes, we take Witten's background of back-reacted $D4$-branes compactified on a thermal circle [5], whose dual, while not pure Yang–Mills, possesses enough similarities (confinement and a mass gap) that we can hope to make generic arguments thereupon. The chief property of this space is its metric G, inducing the following space–time line element where X^μ ($\mu = 0,1,2,3$) are boundary space–time coordinates, τ the compact direction, U the AdS direction and the remaining 4 coordinates parametrise a sphere:

$$ds^2 = g(U)\left(dX^2 + \mathrm{d}\tau^2 f(U)\right) + \frac{1}{g(U)}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$
$$\text{where } g(U) = \left(\frac{U}{R}\right)^{3/2},\ f(U) = 1 - \frac{U_{\mathrm{KK}}^3}{U^3}. \tag{6}$$

It admits a metric singularity at the point $U = U_{KK}$, where the space has a horizon. As is usual, string worldsheets hanging from large loops (of size comparable to the horizon position) will accumulate on the horizon, providing an area-law scaling of their expectation value, rather than a perimeter-law, this is the usual tell-tale sign that confinement has taken place. Then, provided the strings do not have to go very far from the boundary to the horizon, the classical saddle of this action is a string whose geometry is mostly flat, spread out on the horizon itself. By pushing U_{KK} close to infinity, this condition is broadly satisfied, turning the classical saddle of the path integral into a mostly-flat worldsheet. This was the claim proposed in a previous work [3], from which the 4-point function we wish to compute fairly naturally reproduces the Veneziano amplitude as the behaviour associated to flat open-string scattering.

These final steps relied on many broad assumptions, namely that we ignore effects coming from the compact directions (justified through the hierarchy of scales at hand), from the fermionic degrees of freedom (justified by spacetime supersymmetry breaking, and by the insertion of purely bosonic operators), to impose that almost all worldsheets in the sum are heavily accumulating on the horizon, discarding those that do not and ignoring the contribution of the edges of those that do (justified by the near-infinite size of U_{KK}), and to assume no string quantum corrections (both genus/ghost and α'). Our goal is to relax the latter, to allow string tension corrections to the computation at hand, which in physical terms corresponds to letting the string fluctuate around its classical position and experience the U direction curvature and (it will be shown) the compact direction τ.

2.1. Setting up an expansion

From the Polyakov action using the metric shown in Eq. (6), we wish to create a perturbation series for values of the AdS coordinate U close to the horizon position, U_{KK}, which for previously explained reasons should be thought of as a large length scale. The