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Photon mass via current confinement

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ABSTRACT

A parity invariant theory, consisting of two massive Dirac fields, defined in three dimensional space–time, with the confinement of a certain current is studied. It is found that the electromagnetic field, when coupled minimally to these Dirac fields, becomes massive owing to the current confinement. It is seen that the origin of photon mass is not due to any kind of spontaneous symmetry breaking, but only due to current confinement.

1. Introduction

Field theories in three dimensional space time have been a subject of intense study since a couple of decades now. There are several reasons which make such field theories interesting. Firstly often the theories in lower dimensions are simpler than their higher dimensional counterparts. Secondly, it offers new structures like possibility of gauge invariant mass term for gauge field in the form of Chern–Simons term in the action. Interestingly, it was recently found that planar QED with a tree level Chern–Simons term admits a photon which is composite [1]. Theories with Chern–Simons term are found to play an important role in physics of quantum Hall effect and anyonic superconductors [2–6]. Models which exhibit dynamical mass generation and spontaneous chiral symmetry breaking have been constructed and extensively studied [7–11]. In recent years, with the discovery of graphene [12] and topological insulators [13] there has been a renewed interest in the study of lower dimensional field theories.

Colour confinement is one of the still not well understood aspect of QCD. One of the main hindrance is the fact that the low energy dynamics in such theory becomes non-perturbative, which makes dealing with them difficult. To circumvent this difficulty, there have been attempts to assume colour confinement from the beginning and work subsequently to see if one can get some idea about the dynamics of non-Abelian gauge fields [14–16]. In a remarkable paper by Srinivasan and Rajasekaran, it was shown that by assuming quark confinement it was possible to get QCD out of it [16]. Confinement has also been studied in theories defined in three dimensional space–time. It was shown by Polyakov that compact planar QED exhibits charge confinement [17]. While the case of non-compact QED was studied by Grignani et al. [18].

In this paper, it is shown that an assumption of confinement of a certain current gives rise to the photon mass. The theory consider here consists of two species of free Dirac fermions living on the plane, defined such that the theory is even under parity. These fermions are minimally coupled to the photon field. It is found that although the photons in the theory are massive, there is no spontaneous symmetry breaking. It is also shown that when such a theory is defined over a manifold with finite boundary, then there exist massless particles living on the boundary.

In the following section the model is introduced and its various features are discussed. Section 3 deals with the effective action of photon and its mass. Section 4 deals with the case when the theory lives on a manifold with a finite boundary, followed by a brief summary.

2. The model

The Lagrangian describing two species of massive Dirac fermions living in $2+1$ dimensional space–time reads:

$$\mathscr{L}_D = \bar{\psi}_+(i\gamma_+^\mu \partial_\mu - m)\psi_+ + \bar{\psi}_-(i\gamma_-^\mu \partial_\mu - m)\psi_-. \quad (1)$$

Here, gamma matrices are defined for ψ_+ field as $\gamma_+^0 = \sigma_3, \gamma_+^1 = i\sigma_1$ and $\gamma_+^2 = i\sigma_2$. Gamma matrices for ψ_- field are also same as ψ_+ except for γ^2, which is defined as $\gamma_+^2 = -\gamma_-^2$. This deliberate difference in choice of gamma matrices ensures that the Lagrangian is even under parity. It is known that, unlike four

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dimensional space–time, in the three dimensional world parity transformation is defined by reflecting one of the space axis, say Y axis, $(x, y) \rightarrow (x, -y)$. Instead of working with two spinor fields $\psi_\pm$, one can work in a reducible representation by defining $\Psi = (\psi_+, \psi_-)^T$, with $\beta = \gamma^0 = \mathbf{1} \otimes \sigma_3$, $\alpha_1 = \mathbf{1} \otimes \sigma_1$ and $\alpha_2 = \sigma_3 \otimes \sigma_2$, so that the above Lagrangian now reads:

$$\mathscr{L}_D = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi,$$

where $\gamma_{1,2} = \beta\alpha_{1,2}$. Under parity operation, Ψ transforms as $\mathscr{P}\Psi(x, y, t)\mathscr{P}^{-1} = (\sigma_1 \otimes \mathbf{1})\Psi(x, -y, t)$. It can be checked that under this peculiar parity transformation, above Lagrangian remains even despite of having a mass term [19].

As it stands, apart from above mentioned parity transformation, the Lagrangian of this theory is invariant under two independent continuous rigid transformations:

$$\psi_+(r) \rightarrow e^{-i\theta}\psi_+(r),\ \psi_-(r) \rightarrow e^{-i\theta}\psi_-(r); \tag{2}$$

$$\psi_+(r) \rightarrow e^{-i\lambda}\psi_+(r),\ \psi_-(r) \rightarrow e^{i\lambda}\psi_-(r). \tag{3}$$

Here θ and λ are continuous real parameters. These being continuous symmetry operations, give rise to conserved currents as per the Noether theorem:

$$\partial_\mu(j_+^\mu + j_-^\mu) = 0 \text{ and } \partial_\mu(j_+^\mu - j_-^\mu) = 0,$$

where $j^\mu(r) = \bar{\psi}(r)\gamma^\mu\psi(r)$. It turns out that under parity, current $J^\mu = j_+^\mu + j_-^\mu = \bar{\Psi}\gamma^\mu\Psi$ transforms as a vector[1]: $\mathscr{P}J^\mu(x, y, t)\mathscr{P}^{-1} = \Lambda^\mu{}_\nu J^\nu(x, -y, t)$, whereas $\tilde{J}^\mu = j_+^\mu - j_-^\mu = -i\bar{\Psi}\gamma^\mu(\sigma_3 \otimes \sigma_3)\Psi$, transforms as a pseudovector: $\mathscr{P}\tilde{J}^\mu(x, y, t)\mathscr{P}^{-1} = \tilde{J}^\mu(x, -y, t)$. Since the physical photon field transforms as a vector under parity operation: $\mathscr{P}A^\mu(x, y, t)\mathscr{P}^{-1} = \Lambda^\mu{}_\nu A^\nu(x, -y, t)$, its coupling with the current $j_+^\mu + j_-^\mu$ preserves parity while making the symmetry transformation (2) local.

In this paper, we are interested in looking at the physical consequences if the current $j_+^\mu - j_-^\mu$ is confined. As pointed out by Kugo and Ojima in the context of QCD, and further discussed at length in Ref. [21], that the statement of colour charge confinement can be accurately stated as the absence of charge bearing states in the physical sector of the Hilbert space: $Q_{colour}|phys\rangle = 0$. *In what follows, we shall work with a stronger condition than the Kugo–Ojima condition, and demand that the physical space of the theory described by Lagrangian (1) should not have any states which carry* $(j_+^\mu - j_-^\mu)$ *current, that is:* $(j_+^\mu - j_-^\mu)|phys\rangle = 0$.[2] *This shall be referred to as* current confinement condition *henceforth. Since we are demanding a priori that this current confinement condition should hold, it ought to be understood as a constraint. There exists a well known powerful technique to implement such a constraint using what is called the Lagrange multiplier (auxiliary) field [22]. One postulates the existence of a Lagrange multiplier field which is such that its only appearance in the action is via its coupling to the constraint condition. Thus the equation of motion corresponding to this field, obtained by demanding that the functional variation of the action with respect to this field be zero, is simply the constraint condition. It is worth pointing out that such Lagrange multiplier fields have no dynamics of their own, in the sense that there are no terms in the action comprising of spatial or temporal derivatives of these fields to begin with, and their sole purpose of existence is to ensure implementation of the constraint. Thus by enlarging the degree of freedom in the theory by an additional field, one ensures that the constraint condition gets neatly embedded into the action, and hence into the dynamics of the theory. In our case the Lagrange multiplier Bose field is* a_μ, *which is meant to implement the constraint* $(j_+^\mu - j_-^\mu)$, *will only couple to it so that the Lagrangian (1) gets an additional term*:

$$\mathscr{L} = \bar{\psi}_+(i\gamma_+^\mu\partial_\mu - m)\psi_+ + \bar{\psi}_-(i\gamma_-^\mu\partial_\mu - m)\psi_- + a_\mu\left(j_+^\mu - j_-^\mu\right). \tag{4}$$

Note that the equation of motion for a_μ *field:* $\frac{\delta S}{\delta a_\mu} = 0$, *gives the constraint* $j_+^\mu - j_-^\mu = 0$. In order to preserve parity, a_μ field has to be a pseudovector owing to its coupling with pseudovector current. Thus a_μ can in general be written as curl of some vector field χ: $a_\mu = \epsilon_{\mu\nu\lambda}\partial^\nu\chi^\lambda$, and can not have a contribution that can be written as a gradient of some scalar field. This asserts that a_μ can not be a gauge field, since a gauge field under a gauge transformation transforms as a vector $\partial_\mu\Lambda$, which is not consistent with the pseudovector nature of a_μ. Further note that since a_μ is curl of χ_μ, it immediately follows that its divergence vanishes: $\partial_\mu a^\mu = 0$.

In functional integral formulation of quantum field theory, generating functional is an object of central importance, which for this theory reads[3]:

$$Z[\eta_\pm, \bar{\eta}_\pm] = N \int \mathscr{D}[\bar{\psi}_\pm, \psi_\pm, a_\mu]\, e^{iS},$$

where $S = \int d^3x \left(\mathscr{L} + \bar{\eta}_\pm\psi_\pm + \bar{\psi}_\pm\eta_\pm\right)$.

Here η and $\bar{\eta}$ are external sources which are coupled to Fermi fields $\bar{\psi}$ and ψ respectively.

Before we proceed with the details of the quantum theory, it is worth pointing out that if one functionally integrates a_μ *in the above generating functional, one immediately obtains the current confinement condition* $\delta(j_+^\mu - j_-^\mu)$, *since* a_μ *appears linearly in the action. This clearly shows that in the quantum theory as well, the Lagrange multiplier field* a_μ *is properly implementing the current confinement constraint.*

Since the Lagrangian (4) of the theory is invariant under transformation (3), requirement that the generating functional of the theory should also be invariant under (3), that is $\delta Z = 0$, is not unreasonable. Interestingly, it will be seen that this will give rise to Ward–Takahashi identities amongst various n-point functions in this theory and lead to non-trivial consequences. Demanding that Z be invariant under infinitesimal version of transformation (3) means $\delta Z = 0$, which can be written as:

$$\int \mathscr{D}[\bar{\psi}_\pm, \psi_\pm, a_\mu]\,(\delta S)\, e^{iS} = 0.$$

This can further be simplified to read:

$$\int \mathscr{D}[\bar{\psi}_\pm, \psi_\pm, a_\mu]\left(\mp\bar{\eta}_\pm(x)\psi_\pm(x) \pm \bar{\psi}_\pm(x)\eta_\pm(x)\right) e^{iS} = 0. \tag{5}$$

In terms of the generating functional of connected diagrams $W[\bar{\eta}_\pm, \eta_\pm] = -i\ln Z[\bar{\eta}_\pm, \eta_\pm]$, equation (5) becomes:

$$\bar{\eta}_+(x)\frac{\delta W}{\delta\bar{\eta}_+(x)} - \bar{\eta}_-(x)\frac{\delta W}{\delta\bar{\eta}_-(x)} - \eta_+(x)\frac{\delta W}{\delta\eta_+(x)} + \eta_-(x)\frac{\delta W}{\delta\eta_-(x)} = 0. \tag{6}$$

It is often convenient to work with the effective action $\Gamma[\bar{\psi}_\pm, \psi_\pm]$ which is defined to be Legendre transform of $W[\bar{\eta}_\pm, \eta_\pm]$: $W[\bar{\eta}_\pm, \eta_\pm] = \Gamma[\bar{\psi}_\pm, \psi_\pm] + \int d^3x\,(\bar{\eta}_\pm\psi_\pm + \bar{\psi}_\pm\eta_\pm)$, so that equation (6) reads:

[1] Λ is diagonal matrix $\Lambda = \mathrm{diag}(1, 1, -1)$.

[2] The physical space here stands for the set of states in the vector space of the theory, which do not have negative norm [22]. In case when the negative normed states are altogether absent, then the condition $(j_+^\mu - j_-^\mu)|phys\rangle = 0$ holds for the whole of Hilbert space and hence becomes an operator condition $(j_+^\mu - j_-^\mu) = 0$.

[3] Since the current in this theory couples directly to a_μ, it is treated as a dynamical variable instead of χ_μ. Such a treatment is advocated by Hagen in Ref. [23].