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Generalising the coupling between spacetime and matter

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A B S T R A C T

We explore the idea that the coupling between matter and spacetime is more complex than the one originally envisioned by Einstein. We propose that such coupling takes the form of a new fundamental tensor in the Einstein field equations. We then show that the introduction of this tensor can account for dark phenomenology in General Relativity, maintaining a weak field limit compatible with standard Newtonian gravitation. The same paradigm can be applied any other theory of gravitation. We show, as an example, that in the context of conformal gravity a generalised coupling is able to solve compatibility issues between the matter and the gravitational sector.

1. Introduction

One of the most important foundational assumptions of General Relativity (GR) is related with the way in which the curvature of spacetime and the matter in that spacetime interact. In his effort to construct a consistent theory, Einstein [1] chose a proportionality relation between what would be called the Einstein tensor and the stress energy tensor of matter, intended as a continuum. This assumption was suggested by the requirement to have Poisson like equations for the gravitational field, the variational structure of the vacuum equations, but most of all requiring that the newly proposed gravitational field equations would contain naturally the conservation equations for the matter energy continuum.

Almost a hundred years later, the evidence on the evolution of the Universe at cosmological and astrophysical scale has lead the research community to hypothesise the existence of dark (i.e. not interacting electromagnetically) fluids which are characterised by exotic thermodynamics. It is interesting that this dark phenomenology always arises in relation to the gravitational behaviour of collection of many particles (the matter continuum). This is in contrast with the fact that most accurate tests of GR (e.g. the light deflection tests, the perihelion shift of planetary orbits, the Nordtvedt effect [2] and even the recently discovered gravity waves signals [3]) are related to the behaviour of a small number of (test) particles. In other words, so far we have only tested the vacuum Einstein equations. In fact, data on the gravitation of many particle systems always leaves space for an interpretation of data which entails some form of modification of the gravitational interaction, like in the case of inflation [4]. It is difficult to refrain from noting here, in spite of the profound conceptual differences, the resemblance with quantum mechanical systems, in which collections of particles have a global behaviour different for the one of single particle.

It is then possible that, in the same line of thinking, the key to understanding dark phenomenology is related to "global modes" of gravitation of collection of particles? And how could this idea be realised concretely in the framework of relativistic theories of gravitation?

In this letter we propose a possible way to implement such a model. The central idea is that the coupling between matter and spacetime (i.e. the relation between the Einstein tensor and the stress energy tensor) is more complex than a simple law of proportionality and it is described by an additional fundamental tensor. When this idea is applied to pure GR, the generalised coupling is able to generate the non-trivial thermodynamics of the dark fluids without introducing exotic fields. In particular, via reconstruction techniques, we will discover that the new theory solves naturally problems like cosmic acceleration and the flattening of rotation curves of the galaxies, while leaving the GR vacuum phenomenology untouched and retaining the standard Newtonian limit (modulo a rescaling of the gravitational constant). In the case of more complex gravitational theories the use of generalised coupling has other advantages. We will show briefly that, in the case of conformal gravity, the generalised coupling helps resolve issues of compatibility of this theory with non-conformally invariant sources.

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Throughout this paper we will use natural units ($c = 8\pi G = 1$) and Latin indices run from 0 to 3. The conventions are consistent with Wald [5]. The symmetrisation over the indexes of a tensor is defined as $T_{(ab)} = 1/2\,(T_{ab} + T_{ba})$.

2. Modified general relativity

Let us suppose that the interaction between the curvature of spacetime and the stress energy tensor of matter is described by a given tensor field ${\chi_{ab}}^{cd}$ so that the Einstein equations can be written as

$$G_{ab} = {\chi_{ab}}^{cd} T_{cd}\,, \tag{1}$$

where G_{ab} is the Einstein tensor and T_{cd} the matter stress energy tensor. In (1) spacetime couples in a different way with the different thermodynamic properties of matter and we can imagine the components of ${\chi_{ab}}^{cd}$ to represent such coupling. As for the original GR equations, we assume that for the stress energy tensor the usual conservation laws hold $\nabla^c T_{cd} = 0$. Then the Bianchi identities $\nabla^a G_{ab} = 0$ imply

$$\left(\nabla^b {\chi_{ab}}^{cd}\right) T_{cd} + {\chi_{ab}}^{cd} \nabla^b T_{cd} = 0, \tag{2}$$

which can be seen as a propagation equation for ${\chi_{ab}}^{cd}$.

What are the properties of ${\chi_{ab}}^{cd}$? The symmetries of G_{ab} and T_{cd} require

$$\chi_{(ab)(cd)} = \chi_{abcd}. \tag{3}$$

We will also suppose that χ is invertible, i.e. it has no zero eigenvalues and that for a set of free falling shear free vorticity free observers it is diagonal. Since χ_{abcd} represents the coupling between matter and spacetime it is natural to assume that this tensor is different from zero only in presence of matter. The simplest way to implement this condition is to suppose that $\chi_{abcd} = \chi_{abcd}(\mu_0)$ and that $\chi_{abcd}(0) = 0$. As a consequence, if we contract both sides of (1) by the inverse of χ_{abcd}, obtaining

$$\chi^{ab}{}_{cd} G_{ab} = T_{cd} \tag{4}$$

the above equation is only valid if $T_{ab} \neq 0$.

These choices guarantees that the Schwarzschild solution, as well as the other relevant vacuum solutions of General Relativity, is completely unaffected. This fact, in turn, implies that the totality of the classical tests for relativity is automatically satisfied. Other local tests based, for example, on the geodesic deviation, would be irrelevant for the detection of the generalised coupling. In fact, since for a given observer ${\chi_{ab}}^{cd}$ is determined and it contracts the total energy momentum tensor there is no way to reveal its action locally by a single inertial observer.

Things change when we compare the results of gravitational experiments made by different observers. Since ${\chi_{ab}}^{cd}$ is a tensor we can expect different observers to detect different strengths of the gravitational coupling. Such change is the key property one should look for to test (1) against observations. As well known, the only way to perform measurements of a tensorial quantity in a relativistic system is to construct scalar quantities associated to this tensor. In this letter, focus will be given to the simple case of $T_{cd} = \mu u_c u_d$ where u_a ($u_a u^a < 0$) is the four velocity of a given observer. In this case, the R. H. S. of (1) can be written as

$${\chi_{ab}}^{cd} u_c u_d \mu = \left(\chi_1 u_a u_b + \frac{1}{3}\chi_2 h_{ab}\right)\mu \tag{5}$$

where h_{ab} is the projection tensor on the surface orthogonal to u_a and

$$\chi_1 = {\chi_{ab}}^{cd} u^a u^b u_c u_d\,, \tag{6}$$

$$\chi_2 = {\chi_{ab}}^{cd} h^{ab} u_c u_d, \tag{7}$$

are the two scalars, which encode the effect of the generalised coupling in this case. We already know the transformation properties of μ from the structure of T_{cd}, thus the transformation properties (5) and the change of the strength of the gravitational interaction under a Lorentz boost can be understood studying the transformation properties of χ_1 and χ_2.

Upon the passage between two slow moving observers i.e. upon the transformation

$$\tilde{u}_a = \gamma(u_a + v_a)\,, \quad \gamma = (1 - |v|^2)^{-1/2}\,, \quad |v| \ll 1\,, \tag{8}$$

one obtains, for the projection tensor,

$$\tilde{h}_{ab} = h_{ab} + (\gamma^2 - 1)u_a u_b\,, \tag{9}$$

and therefore

$$\tilde{\chi}_1 = \gamma^4 \chi_1 \approx \chi_1\,, \tag{10}$$

$$\tilde{\chi}_2 = \chi_2 + \gamma^2(\gamma^2 - 1)\chi_1 \approx \chi_2\,. \tag{11}$$

Hence if we were to perform an experiment able to measure the gravitational constant in two non-relativistic different frames, the above results tell us that it would not easily reveal changes in the gravitational coupling.

A more efficient testing ground for the generalised coupling is to consider experiments in accelerated frames. Since the velocity of these observers changes in time, the presence of generalised coupling would be evident as a variation of the gravitational coupling with the velocity of the frame. We can sample this effect applying the modified Lorentz transformations to map inertial to non-rotating accelerated observers [6]

$$\tilde{u}_a = \Gamma\left[\left(1 + \dot{u}_b n^b\right) u_a + v_a\right]\,, \tag{12}$$

$$\Gamma = \left[\left(1 + \dot{u}_b n^b\right) - |v|^2\right]^{-1/2}\,, \tag{13}$$

where n_b is the projection of the geodesic deviation of the two observers on the inertial observer rest frame. The transformation laws for the quantities (6) become

$$\tilde{\chi}_1 = \Gamma^4 \left(1 + \dot{u}_b n^b\right)^4 \chi_1\,, \tag{14}$$

$$\tilde{\chi}_2 = \chi_2 + \Gamma^2\left(1 + \dot{u}_b n^b\right)^2\left[\Gamma^2\left(1 + \dot{u}_b n^b\right)^2 - 1\right]\chi_1\,, \tag{15}$$

which now depends on time. In the case of constant acceleration these quantities increase when the velocity increases. Such effect implies that the coupling with gravity will become stronger and stronger as the velocity of the laboratory increases.

The (1) implies that there exist only one frame in which the gravitational field equations look like the standard Einstein equations, the frame in which ${\chi_{ab}}^{cd} = \delta_{(a}{}^{(c}\delta_{b)}{}^{d)}$. We can think of this “Einstein frame” as a special frame for this reason, but in fact, at this level, it has no special characteristics. The main idea that we will examine here is that in the frame in which we make cosmological and astrophysical observation is *not* the Einstein frame and therefore that ${\chi_{ab}}^{cd}$ has a more complex form than the one above.

We now focus on the dynamical aspects of the generalised coupling. In other to understand the properties of dynamical gravitational systems one will have to solve the (1) together with the propagation equation (2) which describes the evolution of χ_{abcd}. At this stage we will not perform a complete analysis of the solutions of this system, but we will rather check if the new theory is able

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