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Testing local Lorentz invariance with short-range gravity

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ABSTRACT

The Newton limit of gravity is studied in the presence of Lorentz-violating gravitational operators of arbitrary mass dimension. The linearized modified Einstein equations are obtained and the perturbative solutions are constructed and characterized. We develop a formalism for data analysis in laboratory experiments testing gravity at short range and demonstrate that these tests provide unique sensitivity to deviations from local Lorentz invariance.

General relativity (GR) is founded on the Einstein equivalence principle, which incorporates local Lorentz invariance, local position invariance, and the weak equivalence principle. GR is known to provide an excellent description of classical gravity over a broad range of length scales. However, modifications of the Einstein equivalence principle associated with local Lorentz violation may arise in an underlying framework compatible with quantum physics such as string theory [1]. Searches for Lorentz violation in gravitational experiments may thus yield clues about the nature of physics beyond GR [2,3].

An important class of precision tests of gravity involves experiments testing its properties at short distances below about a millimeter [4]. Remarkably, even some aspects of the conventional Newton force await verification on this scale, and the presence of larger forces falling as an inverse cubic, quartic, or faster is still compatible with existing experimental data. In this work, we use a comprehensive description of possible deviations from local Lorentz invariance in the pure-gravity sector to study laboratory tests of gravity at short range and to characterize their sensitivity *vis-à-vis* other types of investigations. Our results also provide a formalism for the analysis of data in short-range experiments.

One approach to studying Lorentz violation in gravity is to build a specific model and explore its properties. However, since no compelling signals for Lorentz violation have been uncovered to date, guidance for a broad-based experimental search is perhaps best obtained by developing instead a framework allowing all types of Lorentz violation while including accepted gravitational physics. Effective field theory is one powerful technique along these lines, as it permits a general description of emergent effects from an unobservable scale [5].

In the context of gravity, the effective field theory for Lorentz violation [6] offers a model-independent framework for exploring observables for Lorentz violation. In the pure-gravity sector in Riemann geometry, the action of this theory contains the Einstein–Hilbert action and a cosmological constant along with all coordinate-independent terms involving gravitational-field operators. The pure-gravity action is a subset of the general effective field theory describing matter and gravity known as the gravitational Standard-Model Extension (SME). A term violating Lorentz invariance in the action consists of a Lorentz-violating operator contracted with a coefficient for Lorentz violation that controls the magnitude of the resulting physical effects. It is often convenient to classify the operators according to their mass dimension d in natural units, with operators having larger d likely to induce smaller physical effects at low energies due to a greater suppression by powers of the Newton gravitational constant or, equivalently, by inverse powers of the Planck mass.

To date, comparatively few of the coefficients for Lorentz violation in the pure-gravity sector have been constrained [2]. Most remain unexplored, and some could even involve large Lorentz violation that has escaped detection so far due to "countershading" by feeble couplings [7]. For $d = 4$, certain Lorentz-violating operators generate noncentral orientation-dependent corrections to the inverse-square law. These have been the subject of both theoretical work [8–15] and observation [16–25], and two-sided constraints at various levels down to parts in 10^{11} have been obtained on the nine corresponding coefficients for Lorentz violation. At $d = 6$, many Lorentz-violating operators produce instead corrections to Newton's law involving an inverse *quartic* force [26]. A variety of short-range experiments [27–29] have attained sensi-

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tivities of order 10^{-9} m^2 to the 14 combinations of pure-gravity coefficients controlling this type of Lorentz violation in the nonrelativistic limit, and there are excellent prospects for improved sensitivity [30]. Constraints on some operators of dimensions $d \le 10$ have also been reported, based on the nonobservation of gravitational Čerenkov radiation [31,32] and from data on gravitational waves [33], while proposals for other measurements exist [34–37].

To provide a comprehensive discussion of possible effects of Lorentz violation in the nonrelativistic limit relevant for short-range tests of gravity, we can expand the metric $g_{\mu\nu}$ around the Minkowski spacetime metric $\eta_{\mu\nu}$ and work with the general gauge-invariant and Lorentz-violating Lagrange density $\mathcal{L}$, restricting attention to terms quadratic in the dimensionless metric fluctuation $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ and neglecting the cosmological constant. In this limit, the Einstein–Hilbert term takes the form

$$\mathcal{L}_0 = \tfrac{1}{4}\epsilon^{\mu\rho\alpha\kappa}\epsilon^{\nu\sigma\beta\lambda}\eta_{\kappa\lambda}h_{\mu\nu}\partial_\alpha\partial_\beta h_{\rho\sigma}. \tag{1}$$

Incorporating both Lorentz-violating and Lorentz-invariant operators of arbitrary mass dimension d, the Lagrange density $\mathcal{L}$ can be written as [33]

$$\mathcal{L} = \mathcal{L}_0 + \tfrac{1}{4}h_{\mu\nu}(\hat{s}^{\mu\rho\nu\sigma} + \hat{q}^{\mu\rho\nu\sigma} + \hat{k}^{\mu\nu\rho\sigma})h_{\rho\sigma}. \tag{2}$$

Here, the derivative operators $\hat{s}^{\mu\rho\nu\sigma}$, $\hat{q}^{\mu\rho\nu\sigma}$, and $\hat{k}^{\mu\nu\rho\sigma}$ can be expanded as sums of constant cartesian coefficients $s^{(d)\mu_1\ldots\mu_{d+2}}$, $q^{(d)\mu_1\ldots\mu_{d+2}}$, $k^{(d)\mu_1\ldots\mu_{d+2}}$ for Lorentz violation contracted with factors of derivatives ∂_μ,

$$\begin{aligned}
\hat{s}^{\mu\rho\nu\sigma} &= \sum_{d\ge 4,\ \text{even}} s^{(d)\mu\rho\circ\nu\sigma\circ^{d-3}},\\
\hat{q}^{\mu\rho\nu\sigma} &= \sum_{d\ge 5,\ \text{odd}} q^{(d)\mu\rho\circ\nu\circ\sigma\circ^{d-4}},\\
\hat{k}^{\mu\nu\rho\sigma} &= \sum_{d\ge 6,\ \text{even}} k^{(d)\mu\circ\nu\circ\rho\circ\sigma\circ^{d-5}},
\end{aligned} \tag{3}$$

where a circle index $\circ$ denotes an index contracted into a derivative, and where n-fold contractions are written as $\circ^n$. The operator $\hat{s}^{\mu\rho\nu\sigma}$ is antisymmetric in both the first and second pairs of indices, while $\hat{q}^{\mu\rho\nu\sigma}$ is antisymmetric in the first pair and symmetric in the second, and $\hat{k}^{\mu\nu\rho\sigma}$ is totally symmetric. Contracting any one of these operators with a derivative produces zero. Note that the $d = 4$ piece of $\hat{s}^{\mu\rho\nu\sigma}$ includes a term of the same form as $\mathcal{L}_0$ with an overall scaling factor, which can be set to zero if desired.

In studying the nonrelativistic limit, it is convenient to work with the trace-reversed metric fluctuation

$$\bar{h}_{\mu\nu} = r_{\mu\nu}{}^{\rho\sigma}h_{\rho\sigma}, \tag{4}$$

where

$$r_{\mu\nu}{}^{\rho\sigma} = \tfrac{1}{2}(\eta_\mu{}^\rho\eta_\nu{}^\sigma + \eta_\mu{}^\sigma\eta_\nu{}^\rho - \eta_{\mu\nu}\eta^{\rho\sigma}) \tag{5}$$

is the trace-reverse operator. The modified linearized Einstein tensor obtained by the variation of $\mathcal{L}$ can be written as the sum of the usual linearized Einstein tensor $G_L^{\mu\nu}$ and a correction $\delta G_L^{\mu\nu}$,

$$\begin{aligned}
G_L^{\mu\nu} + \delta G_L^{\mu\nu} &= \tfrac{1}{2}\big(\partial_\rho\partial^{(\mu}\bar{h}^{\nu)\rho} - \eta^{\mu\nu}\partial_\rho\partial_\sigma\bar{h}^{\rho\sigma} - \partial^2\bar{h}^{\mu\nu}\big) + \delta G_L^{\mu\nu}\\
&= -\tfrac{1}{2}\partial^2\bar{h}^{\mu\nu} + \delta G_L^{\mu\nu},
\end{aligned} \tag{6}$$

where in the last line we adopt the Hilbert gauge, $\partial_\mu\bar{h}^{\mu\nu} = 0$. The correction $\delta G_L^{\mu\nu}$ can be expressed as the action of a combination of derivative operators on $\bar{h}^{\mu\nu}$,

$$\delta G_L^{\mu\nu} = \delta\overline{M}^{\mu\nu\rho\sigma}\bar{h}_{\rho\sigma}, \tag{7}$$

where

$$\delta\overline{M}^{\mu\nu\rho\sigma} = \delta M^{\mu\nu\kappa\lambda}r_{\kappa\lambda}{}^{\rho\sigma} \tag{8}$$

with

$$\begin{aligned}
\delta M^{\mu\nu\rho\sigma} &= -\tfrac{1}{4}(\hat{s}^{\mu\rho\nu\sigma} + \hat{s}^{\mu\sigma\nu\rho}) - \tfrac{1}{2}\hat{k}^{\mu\nu\rho\sigma}\\
&\quad - \tfrac{1}{8}(\hat{q}^{\mu\rho\nu\sigma} + \hat{q}^{\nu\rho\mu\sigma} + \hat{q}^{\mu\sigma\nu\rho} + \hat{q}^{\nu\sigma\mu\rho})
\end{aligned} \tag{9}$$

being expressed in terms of the operators appearing in the Lagrange density (2).

The modified linearized Einstein equation takes the form

$$G_L^{\mu\nu} + \delta G_L^{\mu\nu} = 8\pi G_N T^{\mu\nu}, \tag{10}$$

where $T^{\mu\nu}$ is the energy–momentum tensor. The trace-reversed metric fluctuation can be expanded as $\bar{h}^{\mu\nu} = \bar{h}_0^{\mu\nu} + \delta\bar{h}^{\mu\nu}$, where $\bar{h}_0^{\mu\nu}$ is a conventional Lorentz-invariant solution and $\delta\bar{h}^{\mu\nu}$ is the perturbation arising from the correction $\delta G_L^{\mu\nu}$. Solving Eq. (10) at first order then reduces to solving the coupled set of equations

$$\partial^2\bar{h}_0^{\mu\nu} = -16\pi G_N T^{\mu\nu}, \quad \partial^2\delta\bar{h}^{\mu\nu} = 2\delta\overline{M}^{\mu\nu}{}_{\rho\sigma}\bar{h}_0^{\rho\sigma}. \tag{11}$$

In the static limit, the zeroth-order solution satisfies the usual Poisson equation $\nabla^2\bar{h}_0^{\mu\nu} = -16\pi G_N T^{\mu\nu}$ and takes the standard form

$$\bar{h}_0^{\mu\nu}(\boldsymbol{x}) = 4G_N\int d^3x'\,\frac{T^{\mu\nu}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}, \tag{12}$$

while the first-order solution is found to be

$$\delta\bar{h}^{\mu\nu} = 4G_N\,\delta\overline{M}^{\mu\nu}{}_{\rho\sigma}\int d^3x'\,|\boldsymbol{x}-\boldsymbol{x}'|\,T^{\rho\sigma}(\boldsymbol{x}'). \tag{13}$$

Note that this solution is compatible with the Hilbert gauge because $\partial_\mu\delta\overline{M}^{\mu\nu}{}_{\rho\sigma} = 0$.

For applications to short-range experiments, which involve nonrelativistic sources, $T^{\mu\nu}$ is well approximated by its energy-density component $T^{00} = \rho(\boldsymbol{x})$, where $\rho(\boldsymbol{x})$ is the local mass density. We disregard here possible Lorentz-violating modifications to the dispersion relations for various SME matter species [11,38], which generate geodesics on Finsler spacetimes [39,40]. Also, the components of the metric fluctuation can be expressed in terms of a modified gravitational potential $U(\boldsymbol{x})$ producing a modified gravitational acceleration $\boldsymbol{g}(\boldsymbol{x}) = \nabla U$,

$$h^{00} = \tfrac{1}{2}\bar{h}^{00} = 2U, \quad h^{jk} = \tfrac{1}{2}\bar{h}^{00}\delta^{jk} = 2U\delta^{jk}. \tag{14}$$

Expanding $U(\boldsymbol{x}) = U_0(\boldsymbol{x}) + \delta U(\boldsymbol{x})$ as the sum of the usual gravitational potential U_0 and the perturbation δU then yields

$$U_0(\boldsymbol{x}) = G_N\int d^3x'\,\frac{\rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}, \tag{15}$$

as expected. The Lorentz-violating modification to the potential is given by

$$\begin{aligned}
\delta U(\boldsymbol{x}) &= \tfrac{1}{2}\delta h_{00} = \tfrac{1}{2}r_{00\mu\nu}\delta\bar{h}^{\mu\nu}\\
&= 2G_N\delta\overline{\overline{M}}_{0000}\int d^3x'\,|\boldsymbol{x}-\boldsymbol{x}'|\,\rho(\boldsymbol{x}'),
\end{aligned} \tag{16}$$

where for convenience we define the double trace-reversed operator

$$\begin{aligned}
\delta\overline{\overline{M}}_{0000} &= r_{00\mu\nu}r_{00\rho\sigma}\delta M^{\mu\nu\rho\sigma} = \tfrac{1}{4}\delta M^{\rho\rho\sigma\sigma}\\
&= -\tfrac{1}{8}(\hat{s}^{\rho\sigma\rho\sigma} + \hat{k}^{\rho\rho\sigma\sigma}).
\end{aligned} \tag{17}$$

Note the noncovariant traces.