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Generalized gauge U(1) family symmetry for quarks and leptons

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ABSTRACT

If the standard model of quarks and leptons is extended to include three singlet right-handed neutrinos, then the resulting fermion structure admits an infinite number of anomaly-free solutions with just one simple constraint. Well-known examples satisfying this constraint are B–L, L_μ–L_τ, B–$3L_\tau$, etc. We derive this simple constraint, and discuss two new examples which offer some insights to the structure of mixing among quark and lepton families, together with their possible verification at the Large Hadron Collider.

Table 1
Fermion assignments under $U(1)_F$.

Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_F$
$Q_{iL} = (u, d)_{iL}$	3	2	1/6	n_i
u_{iR}	3	1	2/3	n_i
d_{iR}	3	1	−1/3	n_i
$L_{iL} = (\nu, l)_{iL}$	1	2	−1/2	n'_i
l_{iR}	1	1	−1	n'_i
ν_{iR}	1	1	0	n'_i

1. Introduction

In the standard model of particle interactions, there are three families of quarks and leptons. Under its $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, singlet right-handed neutrinos ν_R do not transform. They are not included in the minimal standard model which only has three massless left-handed neutrinos. Since neutrinos are now known to be massive, ν_R should be considered as additions to the standard model. In that case, the model admits a possible new family gauge symmetry $U(1)_F$, with charges $n_{1,2,3}$ for the quarks and $n'_{1,2,3}$ for the leptons as shown in Table 1.

To constrain $n_{1,2,3}$ and $n'_{1,2,3}$, the requirement of gauge anomaly cancellation is imposed. The contributions of color triplets to the $[SU(3)]^2U(1)_F$ anomaly sum up to

$$[SU(3)]^2U(1)_F \ : \ \frac{1}{2}\sum_{i=1}^{3}(2n_i - n_i - n_i); \tag{1}$$

and the contributions of Q_{iL}, u_{iR}, d_{iR}, L_{iL}, l_{iR} to the $U(1)_Y[U(1)_F]^2$ anomaly sum up to

$$U(1)_Y[U(1)_F]^2 \ : \ \sum_{i=1}^{3}\left[6\left(\frac{1}{6}\right) - 3\left(\frac{2}{3}\right) - 3\left(-\frac{1}{3}\right)\right]n_i^2 + \left[2\left(-\frac{1}{2}\right) - (-1)\right]n_i'^2. \tag{2}$$

Both are automatically zero, as well as the $[U(1)_F]^3$ anomaly because all fermions couple to $U(1)_F$ vectorially. The contributions of the $SU(2)_L$ doublets to the $[SU(2)]^2U(1)_F$ anomaly sum up to

$$[SU(2)]^2U(1)_F \ : \ \frac{1}{2}\sum_{i=1}^{3}(3n_i + n'_i); \tag{3}$$

and the contributions to the $[U(1)_Y]^2U(1)_F$ anomaly sum up to

$$[U(1)_Y]^2U(1)_F \ : \ \sum_{i=1}^{3}\left[6\left(\frac{1}{6}\right)^2 - 3\left(\frac{2}{3}\right)^2 - 3\left(-\frac{1}{3}\right)^2\right]n_i + \left[2\left(-\frac{1}{2}\right)^2 - (-1)^2\right]n'_i = \sum_{i=1}^{3}\left(-\frac{3}{2}n_i - \frac{1}{2}n'_i\right). \tag{4}$$

Both are zero if

$$\sum_{i=1}^{3}(3n_i + n'_i) = 0. \tag{5}$$

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Table 2
Examples of models satisfying Eq. (5).

n_1	n_2	n_3	n'_1	n'_2	n'_3	Model
1/3	1/3	1/3	−1	−1	−1	B–L [1]
0	0	0	0	1	−1	L_μ–L_τ [2–5]
1/3	1/3	1/3	0	0	−3	B–$3L_\tau$ [6–9]
1/3	1/3	1/3	3	−3	−3	Ref. [10]
1	1	−2	1	1	−2	Ref. [11]
a	a	$-2a$	0	−1	1	Ref. [12]

Table 3
Two new models satisfying Eq. (5).

n_1	n_2	n_3	n'_1	n'_2	n'_3	Model
1	1	0	0	−2	−4	A
1	1	−1	0	−1	−2	B

There are many specific examples of models which satisfy this condition as shown in Table 2. If there are four families, then $n_{1,2,3} = 1/3$, $n_4 = -1$, and $n'_{1,2,3} = 1$, $n'_4 = -3$, would also satisfy Eq. (5). This may then be considered [13,14] as the separate gauging of B and L.

In this paper, we discuss two new examples which offer some insights to the structure of mixing among quarks and lepton families. Both have nontrivial connections between quarks and leptons. Their structures are shown in Table 3. In both cases, with only one Higgs doublet with zero charge under $U(1)_F$, quark and lepton mass matrices are diagonal except for the first two quark families. This allows for mixing among them, but not with the third family. It is a good approximation to the 3 × 3 quark mixing matrix, to the extent that mixing with the third family is known to be suppressed. In the lepton sector, mixing also comes from the Majorana mass matrix of ν_R which depends on the choice of singlets with vacuum expectation values which break $U(1)_F$. Adding a second Higgs doublet with nonzero $U(1)_F$ charge will allow mixing of the first two families of quarks with the third in both cases. As for the leptons, this will not affect Model A, but will cause mixing in the charged-lepton and Dirac neutrino mass matrices in Model B. Flavor-changing neutral currents are predicted, with interesting phenomenological consequences.

2. Basic structure of Model A

Consider first the structure of the 3 × 3 quark mass matrix $\mathcal{M}_d$ linking $(\bar{d}_L, \bar{s}_L, \bar{b}_L)$ to (d_R, s_R, b_R). Using

$$\Phi_1 = (\phi_1^+, \phi_1^0) \sim (1, 2, 1/2; 0), \tag{6}$$

with $\langle\phi_1^0\rangle = v_1$, it is clear that $\mathcal{M}_d$ is block diagonal with a 2 × 2 submatrix which may be rotated on the left to become

$$\mathcal{M}_d = \begin{pmatrix} c_L & -s_L & 0 \\ s_L & c_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m'_d & 0 & 0 \\ 0 & m'_s & 0 \\ 0 & 0 & m'_b \end{pmatrix}, \tag{7}$$

where $s_L = \sin\theta_L$ and $c_L = \cos\theta_L$. We now add a second Higgs doublet

$$\Phi_2 = (\phi_2^+, \phi_2^0) \sim (1, 2, 1/2; 1), \tag{8}$$

with $\langle\phi_2^0\rangle = v_2$, so that

$$\mathcal{M}_d = \begin{pmatrix} c_L & -s_L & 0 \\ s_L & c_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m'_d & 0 & m'_{db} \\ 0 & m'_s & m'_{sb} \\ 0 & 0 & m'_b \end{pmatrix} \tag{9}$$

is obtained. At the same time, $\mathcal{M}_u$ is of the form

$$\mathcal{M}_u = \begin{pmatrix} m'_u & 0 & 0 \\ 0 & m'_c & 0 \\ m'_{ut} & m'_{ct} & m'_t \end{pmatrix} \begin{pmatrix} c_R & s_R & 0 \\ -s_R & c_R & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{10}$$

where it has been rotated on the right. Because of the physical mass hierarchy $m_u << m_c << m_t$, the diagonalization of Eq. (10) will have very small deviations from unity on the left. Hence the unitary matrix diagonalizing Eq. (9) on the left will be essentially the experimentally observed quark mixing matrix V_{CKM} which has three angles and one phase. Now $\mathcal{M}_d$ of Eq. (9) has exactly seven parameters, the three diagonal masses m'_d, m'_s, m'_b, the angle θ_L, the off-diagonal mass m'_{sb} which can be chosen real, and the off-diagonal mass m'_{db} which is complex. With the input of the three quark mass eigenvalues m_d, m_s, m_b and V_{CKM}, these seven parameters can be determined.

Consider the diagonalization of the real mass matrix

$$\begin{pmatrix} a & 0 & s_1 c \\ 0 & b & s_2 c \\ 0 & 0 & c \end{pmatrix} = V_L \begin{pmatrix} a(1 - s_1^2/2) & 0 & 0 \\ 0 & b(1 - s_2^2/2) & 0 \\ 0 & 0 & c(1 + s_1^2/2 + s_2^2/2) \end{pmatrix} V_R^\dagger, \tag{11}$$

where $s_{1,2} << 1$ and $a << b << c$ have been assumed. We obtain

$$V_L = \begin{pmatrix} 1 - s_1^2/2 & -s_1 s_2 b^2/(b^2 - s_1^2 c^2 - a^2) & s_1 \\ s_1 s_2 a^2/(b^2 + s_2^2 c^2 - a^2) & 1 - s_2^2/2 & s_2 \\ -s_1 & -s_2 & 1 - s_1^2/2 - s_2^2/2 \end{pmatrix}, \tag{12}$$

and

$$V_R^\dagger = \begin{pmatrix} 1 & s_1 s_2 ab/(b^2 - a^2) & -s_1 a/c \\ -s_1 s_2 ab/(b^2 - a^2) & 1 & -s_2 b/c \\ s_1 a/c & s_2 b/c & 1 \end{pmatrix}. \tag{13}$$

Hence

$$V_{CKM} = \begin{pmatrix} c_L & -s_L & 0 \\ s_L & c_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_L, \tag{14}$$

where α is the phase transferred from m'_{db}.

Comparing the above with the known values of V_{CKM} [15], we obtain

$$s_1 = 0.00886, \quad s_2 = 0.0405, \quad s_L = -0.2253, \\ e^{i\alpha} = -0.9215 + i0.3884, \tag{15}$$

with $m_d = m'_d$, $m_s = m'_s$, $m_b = m'_b$ to a very good approximation.

3. Scalar sector of Model A

In addition to $\Phi_{1,2}$, we add a scalar singlet

$$\sigma \sim (1, 1, 0; 1), \tag{16}$$

then the Higgs potential containing $\Phi_{1,2}$ and σ is given by

$$\begin{aligned} V &= m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + m_3^2 \bar{\sigma}\sigma + [\mu\sigma\Phi_2^\dagger\Phi_1 + H.c.] \\ &+ \frac{1}{2}\lambda_1 (\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_3 (\bar{\sigma}\sigma)^2 \\ &+ \lambda_{12} (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda'_{12} (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \lambda_{13} (\Phi_1^\dagger\Phi_1)(\bar{\sigma}\sigma) + \lambda_{23} (\Phi_2^\dagger\Phi_2)(\bar{\sigma}\sigma). \end{aligned} \tag{17}$$