Physics Letters B 766 (2017) 231–237

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Quantization of Yang–Mills theory without the Gribov ambiguity

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ARTICLE INFO

Article history:
Received 17 November 2016
Received in revised form 10 January 2017
Accepted 11 January 2017
Available online 14 January 2017
Editor: M. Cvetič

Keywords:
Yang Mills theory
Quantization
Gribov ambiguity

ABSTRACT

A gauge fixing condition is presented here for non-Abelian gauge theory on the manifold $R \otimes S^1 \otimes S^1 \otimes S^1$. It is proved that the new gauge fixing condition is continuous and free from the Gribov ambiguity. While perturbative calculations based on the new gauge condition behave like those based on the axial gauge in ultraviolet region, infrared behaviours of the perturbative series under the new gauge fixing condition are quite nontrivial. The new gauge condition, which reads $n \cdot \partial n \cdot A = 0$, may not satisfy the boundary condition $A^{\mu}(\infty) = 0$ as required by conventional perturbative calculations for gauge theories on the manifold S^4. However, such contradiction is not harmful for the theory considered here.

1. Introduction

Gauge fixing procedure of non-Abelian gauge theory is a nontrivial issue and hampered by some ambiguities [1,2]. The conventional Faddeev–Popov quantization procedure [3] is based on the equation:

$$\int [\mathcal{D}\alpha(x)] \det(\frac{\delta(G(A))}{\delta\alpha})\delta(G(A)) = 1, \tag{1}$$

where $\alpha(x)$ represents the parameter of gauge transformation, $G(A)$ represents the gauge fixing function and $G(A) = \partial A$ for the Landau gauge. In [1], the author shows that the Landau gauge $\partial \cdot A = 0$ is not a good gauge fixing condition for non-Abelian gauge theories as it does not intersect with each gauge orbit exactly once. Such ambiguity is termed as Gribov ambiguity in literature. In [2], it was proved that there is no continuous gauge fixing condition which is free from the Gribov ambiguity for non-Abelian gauge theory on 3-sphere (S^3) and 4-sphere (S^4) once the gauge group is compact.

The Gribov ambiguity is related to the zero eigenvalues (with nontrivial eigenvectors) of the Faddeev–Popov operator [1,4–6]. It seems natural to work in the so-called Gribov region [1,6], in which the Faddeev–Popov operator is positive definite. The Gribov region is convex and intersects with each gauge orbit at least once [7,8]. Integral region of the gauge potential is restricted to the Gribov region through the no pole condition [1,5], which means that nontrivial poles of propagators of ghosts should vanish in the Gribov region. Such restriction can also be realized through the Gribov–Zwanziger (GZ) action [9–11]. Equivalence between these two methods is proved in [12]. The Gribov region method is extended to general R_ξ gauges in [13] through the field dependent BRST transformation [14,15]. The method can also be extended to the maximal Abelian gauge (see, e.g. Refs. [16–18]).

Although researches based on the GZ action are interesting and fruitful (see, e.g. Refs. [19–23]). There is still Gribov ambiguity even if one works in the Gribov region. A possible solution to the Gribov problem is to work in the absolute Landau gauge [6,4], which is the set of the absolute minima of the functional

$$\int d^4x tr[A^U_\mu(x)A^U_\mu(x)], \tag{2}$$

where U represents an arbitrary gauge transformation. It is, however, difficult to perform analytical calculations in this gauge. An alternative way is to average over Gribov copies as in [24,25], which avoids the Neuberger zero problem of the standard Faddeev–Popov quantization procedure. One may also take an extra constraint introduced in [26,27] that eliminates infinitesimal Gribov copies without the geometric approach.

For an algebraic gauge condition like the axial gauge $n \cdot A = 0$, the degeneracy is independent of the gauge potential. It seems that calculations in such gauge are not affected by the Gribov ambiguity. However, such gauge condition is not continuous for gauge theories on the manifold S^4. To see this, we consider the equation:

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http://dx.doi.org/10.1016/j.physletb.2017.01.016

$$Un\cdot AU^{\dagger}+\frac{i}{g}Un\cdot\partial U^{\dagger}=0\Rightarrow$$
$$U^{\dagger}(x)\propto P\exp(ig\int_{-\infty}^{0}n\cdot A(x+sn^{\mu})). \quad (3)$$

It is impossible to choose the proportional function in above equation so that $U(x)$ takes unique value at infinity. This is in contradiction with the continuity of gauge transformations as the infinity is an ordinary point on S^4. There is another famous algebraic gauge termed as the space-like planar gauge [28–31], in which the gauge fixing term reads

$$\mathcal{L}_{\text{fix}}\equiv-\frac{1}{n^2}tr[n\cdot A\partial^2 n\cdot A], \quad (4)$$

where n^{μ} is a space like vector. The space-like planar gauge is free from the Gribov ambiguity [30] and not continuous for gauge theories on S^4.

In this paper, we consider non-Abelian gauge theory on the 3 + 1 dimensional manifold $R\otimes S^1\otimes S^1\otimes S^1$. Topological properties of the manifold are interesting and may be related to confinement of quarks as displayed in [32]. Gauge potentials on the manifold $R\otimes S^1\otimes S^1\otimes S^1$ satisfy the periodic boundary conditions,

$$A^{\mu}(t,x_1+L_1,x_2,x_3)=A^{\mu}(t,x_1,x_2,x_3)$$
$$A^{\mu}(t,x_1,x_2+L_2,x_3)=A^{\mu}(t,x_1,x_2,x_3)$$
$$A^{\mu}(t,x_1,x_2,x_3+L_3)=A^{\mu}(t,x_1,x_2,x_3), \quad (5)$$

where L_i $(i=1,2,3)$ are large constants. It is hard to maintain Lorentz invariance in theories on the manifold. We do not consider such defect here. We will show that the gauge condition

$$n\cdot\partial n\cdot A=0 \quad (6)$$

is continuous and free from the Gribov ambiguity for gauge theories on the manifold $R\otimes S^1\otimes S^1\otimes S^1$, where n^{μ} represents directional vectors along x_i-axis $(i=1,2,3)$. We can rewrite the gauge condition in momentum space, which reads,

$$n\cdot A(k)=0\ (\text{for}\ n\cdot k\neq 0). \quad (7)$$

We see that the gauge fixing condition is equivalent to the axial gauge for $n\cdot k\neq 0$.

The paper is organized as follows. In Sec. 2, we describe gauge theory on $R\otimes S^1\otimes S^1\otimes S^1$ briefly. In Sec. 3, we consider non-Abelian gauge theory on $R\otimes S^1\otimes S^1\otimes S^1$ and present the proof that the gauge condition (6) is continuous and free from the Gribov ambiguity. In Sec. 4, we discuss propagators of gluons under the new gauge fixing condition. Our conclusions are presented in Sec. 5.

2. Gauge theories on $R\otimes S^1\otimes S^1\otimes S^1$

In this section, we describe gauge theories on the manifold $R\otimes S^1\otimes S^1\otimes S^1$. The manifold $R\otimes S^1\otimes S^1\otimes S^1$ can be obtained from the Minkowski space through the identification

$$(t,x_1,x_2,x_3)\sim(t,x_1+L_1,x_2,x_3)\sim(t,x_1,x_2+L_2,x_3)$$
$$\sim(t,x_1,x_2,x_3+L_3), \quad (8)$$

where L_i $(i=1,2,3)$ are large constants. We take the following periodic boundary conditions,

$$A^{\mu}(t,x_1+L_1,x_2,x_3)=A^{\mu}(t,x_1,x_2,x_3)$$
$$A^{\mu}(t,x_1,x_2+L_2,x_3)=A^{\mu}(t,x_1,x_2,x_3)$$
$$A^{\mu}(t,x_1,x_2,x_3+L_3)=A^{\mu}(t,x_1,x_2,x_3) \quad (9)$$

in this paper. Effects of the center vortexes like those shown in [33] are not considered here. We require that

$$U(t,x_1+L_1,x_2,x_3)=U(t,x_1,x_2,x_3)$$
$$U(t,x_1,x_2+L_2,x_3)=U(t,x_1,x_2,x_3)$$
$$U(t,x_1,x_2,x_3+L_3)=U(t,x_1,x_2,x_3), \quad (10)$$

for continuous gauge transformation on the manifold $R\otimes S^1\otimes S^1\otimes S^1$.

Quantum field theories on $R\otimes S^1\otimes S^1\otimes S^1$ are quite similar to quantum mechanics in the box normalization scheme. In such scheme the momentum operator $-i\vec{\nabla}$ is a Hermitian operator as surface terms vanish according to periodic boundary conditions. For quantum field theories on $R\otimes S^1\otimes S^1\otimes S^1$, the surface terms also vanish according to periodic conditions (9). Thus the operator $-i\vec{\nabla}A^{\mu}$ is Hermitian. We can get perturbative series similar to those in quantum field theory on S^4.

We should emphasize here that the manifold $R\otimes S^1\otimes S^1\otimes S^1$ is not Lorentz invariant, which seems troublesome. The manifold $R\otimes S^1\otimes S^1\otimes S^1$ looks like the Minkowski space locally. It seems to us that the Lorentz invariance can be restored for local quantities in the limit $L_i\to\infty$ $(i=1,2,3)$. In fact, Feynman rules of quantum theories on the manifold $R\otimes S^1\otimes S^1\otimes S^1$ are similar to those on the manifold R^4 except for that momenta of particles take discrete values for the theory considered here. For the case that $L_i\to\infty$ $(i=1,2,3)$, summations over discrete momenta values tend to integrals over the momenta space once such integrals are not affected by ultraviolet divergences or mass singularities. In perturbative calculations, ultraviolet divergences are absorbed into physical constants through renormalization procedures. Mass singularities are harmless for local quantities once the summation over all possible initial and final states has been performed according to the famous Kinoshita–Lee–Nauenberg (KLN) theorem [34, 35]. As a result, we simply assume that the Lorentz invariance can be restored in the limit $L_i\to\infty$ $(i=1,2,3)$ for local quantities which are multiplicative renormalized and infrared safe. Renormalization properties and KLN cancellations of theories on the manifold $R\otimes S^1\otimes S^1\otimes S^1$ are not considered here.

To explain what happens on the manifold $R\otimes S^1\otimes S^1\otimes S^1$, we consider a gauge theory of which the gauge group is $U(1)$. Although such gauge theory is free from the Gribov ambiguity in Landau gauge, it is convenient to take this theory as an example to show that the gauge condition

$$n\cdot\partial n\cdot A=0 \quad (11)$$

is a continuous gauge on $R\otimes S^1\otimes S^1\otimes S^1$ and free from the Gribov ambiguity, where n^{μ} is the directional vector along x_i-axis $(i=1,2,3)$. According to the boundary conditions (5), we can write $n\cdot A(x)$ as:

$$n\cdot A(x)=\sum_{m}e^{i2\pi m\frac{n\cdot x}{n\cdot L}}f_m(x_T), \quad (12)$$

where $L^{\mu}=(0,L_1,L_2,L_3)$ and x_T is defined as

$$x_T^{\mu}\equiv x^{\mu}-\frac{n\cdot x}{n^2}n^{\mu}. \quad (13)$$

A continuous gauge transformation $U(x)=\exp(i\phi(x))$ should also satisfy the boundary conditions (5). We thus have:

$$\phi(x)=2\pi N\frac{n\cdot x}{n\cdot L}+\sum_{m\neq 0}e^{i2\pi m\frac{n\cdot x}{n\cdot L}}g_m(x_T), \quad (14)$$

where N is an arbitrary integer. Under the gauge transformation $U(x)$, we have