ARTICLE IN PRESS

[Physics Letters B](http://dx.doi.org/10.1016/j.physletb.2016.11.058) ••• (••••) •••-•••

1 Contents lists available at [ScienceDirect](http://www.ScienceDirect.com/) 66 Physics Letters B 4 69 2 собраните при представите на представите на селото на 3 68 5 70 6 1999 - 19

11 II outpon thousand displacement on fourth only growth. $\frac{1}{12}$ Horizon thermodynamics in fourth-order gravity

13 and the contract of the con 14 Meng-Sen Ma^{a,b,}∗⁷⁹

15 80 ^a *Department of Physics, Shanxi Datong University, Datong 037009, China*

16 81 ^b *Institute of Theoretical Physics, Shanxi Datong University, Datong 037009, China* 17 \sim

¹⁹ ARTICLE INFO ABSTRACT ⁸⁴

20 and the state of *Article history:* Received 5 September 2016 Received in revised form 25 November 2016 Accepted 29 November 2016 Available online xxxx Editor: M. Cvetič

21 Article history: **Example 21** Article history: **Example 20** In the framework of horizon thermodynamics, the field equations of Einstein gravity and some other 86 22 Beconder a second-order gravities can be rewritten as the thermodynamic identity: *dE* = *TdS* − *PdV*. However, in 87
Possived in ravised form 25 November 2016 23 Received in revised form 25 November 2016 **order to construct the horizon thermodynamics in higher-order gravity, we have to simplify the field 88** 24 Australia 2008 in this paper, we study the fourth-order gravity and convert it to second-order gravity age 25 Faither March 2008. We are the so-called "Legendre transformation" at the cost of introducing two other fields besides the metric 90 26 91 field. With this simplified theory, we implement the conventional procedure in the construction of the 27 27 27 22 and 2 and 2 and 3 and 4 dimensional spacetime. We find that the field equations in the 28
approach to derive the same black hole mass as that by other methods. fourth-order gravity can also be written as the thermodynamic identity. Moreover, we can use this

29 \degree \degree \degree 2016 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license \degree $\rm (http://creativecommons.org/licenses/by/4.0/">(http://creativecommons.org/licenses/by/4.0/">(http://creativecommons.org/licenses/by/4.0/).}) The image is a 95000.$ $\rm (http://creativecommons.org/licenses/by/4.0/">(http://creativecommons.org/licenses/by/4.0/">(http://creativecommons.org/licenses/by/4.0/).}) The image is a 95000.$ $\rm (http://creativecommons.org/licenses/by/4.0/">(http://creativecommons.org/licenses/by/4.0/">(http://creativecommons.org/licenses/by/4.0/).}) The image is a 95000.$

34 **1. Introduction 1. Introduction 1. Introduction 1999 1. Introduction 1999 1.** Introduction **1999 1.** Int **1. Introduction**

³⁶ 11 that long been known that gravitational system has thermody, means that in the field equations there are at most the second- $\frac{37}{2}$ is the super-trace integrative derivatives of metric functions. In fact, one can find that in $\frac{102}{2}$ $\frac{38}{103}$ indices only the first-order the first-order derivatives of metric functions ex-
In the conventional thermodynamic systems black holes also many cases only the first-order derivatives of metric functions ex- $\frac{39}{104}$ Just the conventional incrimedy nature systems, back notes also $\frac{39}{104}$ ist. The work [\[23\]](#page--1-0) on the Hořava–Lifshitz theory is the first study $\frac{104}{104}$ $\frac{40}{105}$ investing horizon thermodynamics in higher-order derivative gravity. But $\frac{105}{105}$ in horizon thermodynamics in higher-order derivative gravity. But $\frac{105}{105}$ $\frac{41}{100}$ it is shown that the field equations of $\frac{106}{100}$.
Not only that it was found that the field equations of Finstein it is shown that the field equations of Hořava–Lifshitz gravity in $\frac{42}{12}$ interval that, it was found that the field equations of Emstern
 $\frac{42}{12}$ ravity and other more general gravitational theories such as $f(R)$ the static, spherically symmetric case, only include the first-ord $\frac{43}{10}$ gravity and other more general gravitational theories, such as $f(x)$ derivatives of metric functions. Generally, in higher derivative grav-44 Blavity, can be derived from an equation of state of focal spacetime
Ities, the field equations are full of higher-order derivatives of met-It has long been known that gravitational system has thermodynamic properties since the works of Hawking and Bekenstein [\[1,2\].](#page--1-0) Just like conventional thermodynamic systems, black holes also have the temperature, entropy and other thermodynamic quantities. Besides, black holes also have fruitful phase structures [\[3–16\].](#page--1-0) Not only that, it was found that the field equations of Einstein gravity and other more general gravitational theories, such as *f (R)* gravity, can be derived from an equation of state of local spacetime thermodynamics [\[17,18\].](#page--1-0)

46 11¹¹ There is another route to explore the relationship between the 111 directions and are very compinented, we cannot uncerty extend ₄₇ gravitational system and its relevant thermodynamic properties. It the previous approach to these theories, we should first require the theories of the t 48 is the framework of horizon thermodynamics proposed by Pad-
⁴⁸ ¹¹³ $_{49}$ manabhan [\[19\].](#page--1-0) It is shown that Einstein's field equations for a cress can be done via a regentive transformation [25–27]. One can $_{114}$ $_{50}$ spherically symmetric spacetime can be written in the form of $_{\rm{even}}$ even convert the inglier-derivative gravity from the original jordan $_{\rm{115}}$ 51 116 thermodynamic identity: *dE* = *TdS* − *PdV* . This makes the connec- 52 tion between gravity and thermodynamics more closely. The radial of their redefinition [30]. Under some conditions, one can verify 117 53 pressure *P* is the $\binom{r}{r}$ component of energy–momentum tensor. This the equivalence of black hole thermodynamics between the two 118 54 approach has also been extended to the non-spherically symmet- and aller subsetedimeters we do not want to dear with the non-55 ric cases [\[20,21\]](#page--1-0) and other theories of gravity, such as Lovelock the and infinitious or inglier-derivative gravity in the Einstein 120 56 gravity [\[22\],](#page--1-0) Hořava–Lifshitz theory [\[23\]](#page--1-0) and Einstein gravity with than because there is suif no consensus on the physical equivconformal anomaly [\[24\].](#page--1-0)

E-mail addresses: mengsenma@gmail.com, [ms_ma@sxdtdx.edu.cn.](mailto:ms_ma@sxdtdx.edu.cn)

 $\frac{45}{10}$ incritions anticle (17,10).
In There is another route to explore the relationship between the still functions and are very complicated. We cannot directly extend 57 conformal anomaly [24]. **Example 2018** alence between the Jordan frame and the Einstein frame [\[32–34\].](#page--1-0) ₁₂₂ 58 123 In this paper we will study a fourth-order derivative gravity. Via 59 124 the "Legendre" transformation, it can be reduced to a second-order 60 125 * Correspondence to: Department of Physics, Shanxi Datong University, Datong 62 62 62 62 E-mail addresses: mengsenma@gmail.com, ms_ma@sxdtdx.edu.cn. **Example 1978** this way, the field equations can be simplified greatly. Thus, we 127 the previous approach to these theories. We should first reduce the higher-derivative gravity to some lower-derivative gravity. This process can be done via a "Legendre" transformation [\[25–27\].](#page--1-0) One can even convert the higher-derivative gravity from the original Jordan frame into Einstein frame by a conformal transformation [\[28,29\]](#page--1-0) or field redefinition [\[30\].](#page--1-0) Under some conditions, one can verify the equivalence of black hole thermodynamics between the two frames [\[30,31\].](#page--1-0) However, we do not want to deal with the horizon thermodynamics of higher-derivative gravity in the Einstein frame, because there is still no consensus on the physical equivderivative gravity with some additional auxiliary fields, which is

^{61 037009} China **Still equivalent to the original fourth-order derivative gravity.** In 126 037009, China

⁶³ 128 <http://dx.doi.org/10.1016/j.physletb.2016.11.058>

^{64 0370-2693/© 2016} The Author. Published by Elsevier B.V. This is an open access article under the CC BY license [\(http://creativecommons.org/licenses/by/4.0/](http://creativecommons.org/licenses/by/4.0/)). Funded by 129 65 130 SCOAP3.

TICLE IN PRE JID:PLB AID:32462 /SCO Doctopic: Theory [m5Gv1.3; v1.191; Prn:2/12/2016; 12:03] P.2 (1-6)

2 *M.-S. Ma / Physics Letters B* ••• *(*••••*)* •••*–*•••

¹ can extract the useful information from the field equations to con- $S = S_C + S_M = \int d^d x (C_C + C_M)$ 2 Struct the horizon thermodynamics. $\qquad \qquad J$ and $\qquad \qquad$ 57 ST struct the horizon thermodynamics.

³ The plan of this paper is as follows: In Sec. 2 we give a very the second left of the second second that $\int_{\mathcal{A}} d\mu = \int_{\mathcal{A}} d\mu = \int_{\mathcal{A}} d\mu$ 4 short introduction to horizon thermodynamics in Einstein gravity. $- \int u \wedge \sqrt{-g} (L_G + L_M)$, (3.1) 5 70 We present the necessary demonstrations on some notations. In 6 Sec. 3 we introduce the fourth-derivative gravity theory and obtain 6 Where S_M represents the action of matter fields, and the gravita- 71 7 the second-derivative gravity via the "Legendre" transformation. In tional Lagrangian L_G takes the form \overline{L} 8 Sec. [4](#page--1-0) we give some examples to show the horizon thermodynam-⁹ ics in fourth-derivative gravity in 3 and 4 dimensional spacetime. $L_G = -(R - 2\Lambda + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu})$, (3.2) ⁷⁴ ¹⁰ In Sec. [5](#page--1-0) we summarize our results and discuss the possible future \overline{K} ¹¹ directions. In Appendix, we give the complete form of some field with $\kappa = 16\pi G_d$. It should be noted that the matter fields are necequations in components.

16 81 In this section, we simply introduce the horizon thermodynam-17 ics in Einstein gravity first proposed in [\[19\].](#page--1-0) For a static, spherically $9\mu v + E\mu v = 8\pi G dL\mu v$, (3.3) 82 ¹⁸ symmetric spacetime, the metric in the Schwarzschild gauge can where the structure of the structure of the schwarzschild gauge can where the schwarzschild gauge of the schwarzschild gauge can where the schwarzschild g 19 be written as the contract of the contract be written as

$$
ds^{2} = -f(r)dt^{2} + f(r)^{-1}dr^{2} + r^{2}d\Omega^{2}.
$$
\n(2.1)
$$
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu},
$$
\n
$$
g_{\mu\nu} = g_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu},
$$

$$
{}_{25}^{24} \t G^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} - \frac{1}{2} R g^{\mu}{}_{\nu} = 8 \pi T^{\mu}{}_{\nu},
$$
\n
$$
{}_{22}^{24} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box R_{\mu\nu} + \nabla_{\rho} \nabla_{\sigma} R^{\rho \sigma} g_{\mu\nu} - 2 \nabla_{\rho} \nabla_{(\mu} R_{\nu)}{}^{\rho})
$$
\n
$$
{}_{20}^{20} \t H^{\beta} (\Box
$$

one can obtain

 $r_{+} f'(r_{+}) - 1 = 8\pi r_{+}^{2}$ $\frac{2}{+}P$, (2.3)

and thus

$$
\frac{31}{32} \quad d\left(\frac{r_+}{2}\right) = \frac{f'(r_+)}{4\pi}d(\pi r_+^2) - PdV,
$$
\n(2.4) field equations. We can introduce two conjugate fields in the following way:

\n
$$
\frac{31}{2} \quad d\left(\frac{r_+}{2}\right) = \frac{f'(r_+)}{4\pi}d(\pi r_+^2) - PdV,
$$

34 where r_+ represents the position of the event horizon, which must $\delta \mathcal{L}$ $\delta \mathcal{L}$ $\delta \mathcal{L}$ $\delta \mathcal{L}$ 35 satisfy $f(r_+) = 0$. $P = T^r r |_{r=r_+}$, is the radial pressure of matter at $\Phi = \frac{3\pi}{8R}$, $\Psi^{\mu\nu} = \frac{3\pi}{8R}$. 36 the horizon. $V = 4\pi r_+^3/3$ is called the "areal volume". According $v_0 = 0.64 \mu v_0$ 37 to Eq. (2.1), it is just the volume of the black hole with horizon where $\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M$. 38 radius r_+ in the coordinate.
We can further set $\sqrt{-g}\phi = \Phi$ and $\sqrt{-g}\psi^{\mu\nu} = \Psi^{\mu\nu}$. In this 103 radius r_{+} in the coordinate.

$$
T = \frac{\kappa}{2\pi} = \frac{f'(r_+)}{4\pi},
$$
\n
$$
\phi = \frac{1}{\kappa}(1 + 2\alpha R), \quad \psi^{\mu\nu} = \frac{2\beta}{\kappa}R^{\mu\nu}.
$$
\n(3.6)

43 Eq. (2.4) is just the conventional thermodynamic identity $dE =$ Now we take the "Legendre" transformation according to the two 108 $T dS - P dV$ with $E = r_+/2$, $S = A/4 = \pi r_+^2$. In the source-free case, pairs of conjugated quantities. First, we should invert Eq. (3.6) to ⁴⁵ the metric function represents Schwarzschild black hole. For this optrain R and R_{app} as functions of ϕ and $\psi_{\mu\nu}$ respectively. This black hole, *E* is just the mass *M* of the black hole.

⁴⁷ This result above only depends on the theories of gravity under $\frac{1}{12}$ inition: ⁴⁸ consideration. It has nothing to do with the concrete black hole **that is a set of the contract of the concrete** black hole 49 solution. The contributions from matter fields have been contained $\mathcal{H}(\phi_1|\psi_{\ldots}) = \Phi R + \Psi^{\mu\nu}R_{\ldots} - \mathcal{L}$ erally.

52 In this case, only two pairs of thermodynamic variables exist, $\begin{array}{ccc} 52 & 4\alpha\kappa & 4\beta^{\gamma} & \mu\kappa & \kappa \end{array}$ (117) 53 which are the intensive quantities (T, P) and the extensive quan-54 119 tities *(S, V)*. In this framework the thermodynamic properties are ⁵⁵ directly related to the gravitational theories under consideration. At last, we define the state of the state of $^{12\text{C}}$ 56 121 The details of matter content are not important and the concrete ⁵⁷ black hole solutions are also not necessary. In this framework, we $\mathcal{L}_H(g_{\mu\nu}, \phi, \psi_{\mu\nu}) = \Phi R + \Psi^{\mu\nu} R_{\mu\nu} - \mathcal{H}(\phi, \psi_{\mu\nu})$ ⁵⁸ have studied the phase transitions and thermodynamic stabilities \overline{a} and \overline{a} \overline{a} and \overline{a} and 59 of black holes in general relativity and Gauss–Bonnet gravity [\[35\].](#page--1-0) $= \sqrt{-g} \int \phi R + \psi^{\mu\nu} R_{\mu\nu} - \frac{24A}{\mu} - \frac{(\kappa \psi - 1)^2}{2}$

63 In this section we will generalize the original horizon thermo-⁶⁴ dynamics approach to the higher-derivative gravity. Let us consider Treating $g_{\mu\nu}$, ϕ , $\psi_{\mu\nu}$ as three independent field variables, the vari-
⁶⁴ a fourth-derivative gravity action

$$
S = S_G + S_M = \int d^d x \, (\mathcal{L}_G + \mathcal{L}_M)
$$

$$
=\int d^d x \sqrt{-g} \left(L_G + L_M \right),\tag{3.1}
$$

where S_M represents the action of matter fields, and the gravitational Lagrangian *LG* takes the form

$$
L_G = \frac{1}{\kappa} \left(R - 2\Lambda + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} \right),\tag{3.2}
$$

⁷⁷ equations in components. The same of the section of the section of the horizon thermodynamics, although its concrete $\frac{77}{2}$ 13 78 form is not necessary. We need energy–momentum tensor to de-14 79 **2. Horizon thermodynamics in Einstein gravity** with $\kappa = 16\pi G_d$. It should be noted that the matter fields are nectermine the *PdV* term uniquely.

15 80 The field equations that follow from the action Eq. (3.1) are

$$
\mathcal{G}_{\mu\nu} + E_{\mu\nu} = 8\pi G_d T_{\mu\nu},\tag{3.3}
$$

where

$$
ds^{2} = -f(r)dt^{2} + f(r)^{-1}dr^{2} + r^{2}d\Omega^{2}.
$$
\n(2.1)
$$
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu},
$$
\nSubstituting the metric into Einstein field equation\n
$$
E_{\mu\nu} = 2\beta(R_{\mu\rho}R_{\nu}{}^{\rho} - R^{\rho\sigma}R_{\rho\sigma}g_{\mu\nu}) + 2\alpha R(R_{\mu\nu} - Rg_{\mu\nu})
$$
\n
$$
G_{\mu\nu} = 2\beta(R_{\mu\rho}R_{\nu}{}^{\rho} - R^{\rho\sigma}R_{\rho\sigma}g_{\mu\nu}) + 2\alpha R(R_{\mu\nu} - Rg_{\mu\nu})
$$
\n(2.2)

$$
+2\alpha \left(g_{\mu\nu} \Box R - \nabla_{\mu} \nabla_{\nu} R\right). \tag{3.4}
$$

²⁷ 1f substituting the metric (2.1) into these field equations, the ex-²⁸ $r_+f'(r_+)-1=8\pi r_+^2 P$, (2.3) pressions are so complicated that one cannot directly construct the ²⁹ and thus and the settlement of the se

30 and thus the strength of the strength of the S₅ Now we employ the "Legendre" transformation to simplify the ⁹⁵ lowing way:

$$
\Phi = \frac{\delta \mathcal{L}}{\delta R}, \quad \Psi^{\mu \nu} = \frac{\delta \mathcal{L}}{\delta R_{\mu \nu}}, \tag{3.5}
$$

where $\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M$.

39 39 239 104 Considering the temperature of the black hole is the state of the way, we can obtain way, we can obtain

$$
T = \frac{\kappa}{2\pi} = \frac{f'(r_+)}{4\pi},
$$
\n
$$
\phi = \frac{1}{\kappa}(1 + 2\alpha R), \quad \psi^{\mu\nu} = \frac{2\beta}{\kappa}R^{\mu\nu}.
$$
\n(3.6)

⁴⁶ black hole, *E* is just the mass *M* of the black hole. $\frac{1}{11}$ can be easily done. Then substituting them into the following def-Now we take the "Legendre" transformation according to the two pairs of conjugated quantities. First, we should invert Eq. (3.6) to obtain *R* and $R_{\mu\nu}$ as functions of ϕ and $\psi_{\mu\nu}$, respectively. This inition:

\n The contributions from matter fields have been contained in the pressure *P*. Obviously, except for vacuum cases,
$$
E \neq M
$$
 and $\mathcal{H}(\phi, \psi_{\mu\nu}) = \Phi R + \Psi^{\mu\nu} R_{\mu\nu} - \mathcal{L}$ \n

\n\n In this case, only two pairs of thermodynamic variables exist, which are the intensive quantities (T, P) and the extensive quantities are (T, P) and the extensive quantities are (T, P) and the extensive quantities are (T, P) .\n

\n\n The equations are the inverse equations, $E \neq M$ and $\psi_{\mu\nu} = \sqrt{-g} \left[\frac{(\kappa \phi - 1)^2}{4\alpha \kappa} + \frac{\kappa}{4\beta} \psi^{\mu\nu} \psi_{\mu\nu} + \frac{2\Lambda}{\kappa} - L_M \right].\n$

\n\n The solutions are the inverse quantities (T, P) and the extensive quantities are (3.7) .\n

At last, we define

60 125 61 126 **3. The fourth-order gravity** ^L*^H (gμν ,φ,ψμν)* ⁼ *^R* ⁺ *μν ^Rμν* [−] ^H*(φ,ψμν)* ⁼ √−*^g ^φ^R* ⁺ *^ψμν ^Rμν* [−] ² *^κ* [−] *(κφ* [−] ¹*)*² 4*ακ*

62 127 − *κ* ⁴*^β ^ψμνψμν* ⁺ *LM .* (3.8)

⁶⁵ a fourth-derivative gravity action **the Case of the Lagrangian (3.8)** yields the following field equations: ¹³⁰ Treating $g_{\mu\nu}$, ϕ , $\psi_{\mu\nu}$ as three independent field variables, the vari-

Please cite this article in press as: M.-S. Ma, Horizon thermodynamics in fourth-order gravity, Phys. Lett. B (2016), http://dx.doi.org/10.1016/j.physletb.2016.11.058

Download English Version:

<https://daneshyari.com/en/article/5495068>

Download Persian Version:

<https://daneshyari.com/article/5495068>

[Daneshyari.com](https://daneshyari.com)