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Phantom solution in a non-linear Israel–Stewart theory

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ABSTRACT

In this paper we present a phantom solution with a big rip singularity in a non-linear regime of the Israel–Stewart formalism. In this framework it is possible to extend this causal formalism in order to describe accelerated expansion, where assumption of near equilibrium is no longer valid. We assume a flat universe filled with a single viscous fluid ruled by a barotropic EoS, $p = \omega\rho$, which can represent a late time accelerated phase of the cosmic evolution. The solution allows to cross the phantom divide without evoking an exotic matter fluid and the effective EoS parameter is always lesser than -1 and constant in time.

1. Introduction

It is well known that along the evolution of our Universe are involved a great number of dissipative processes, however, there is not a definitive model to describe the overall dynamics of this dissipation since some explanations are based on speculative physics [1]. In the search for a coherent model, was found that these dissipative processes can be well treated by employing a relativistic theory of bulk viscosity. Qualitatively, the bulk viscosity can be interpreted as a macroscopic consequence coming from the frictional effects in mixtures. The dark sector of the universe is the main material sector and is the one that presents more open questions about its real nature, in terms of what we know from our current theories. In this sense, one of the possible reasonable assumptions is to consider unified dark matter models, where the dark matter exhibits dissipative effects, which can lead the accelerated expansion that is associated to the dark energy component. For an inhomogeneous and isotropic universe only the bulk viscosity is present, and in general it is assumed that is ruled by a simple suitable expression in terms of the energy density of the dark fluid. A causal and stable theory of thermal phenomena in the presence of gravitational fields, was introduced by Israel and Stewart (IS) [2], it provided a better description than Eckart and Landau and Lifshitz theories, but sharing with them the characteristic that only small deviations from equilibrium are assumed, then the transport equation is linear in the bulk viscous pressure; despite the improvements in the thermal description obtained with the IS theory, it was found in Ref. [1] that for an inflating universe driven by viscosity it was necessary to consider a non-linear extension of the theory. This extension was done in [3], and the major aim of this work was to study inflationary solutions with no need of scalar fields, this new theory includes the IS theory in the linear regime ensuring the causality and stability, obeys the second law of thermodynamics, i.e., the positivity of the entropy production is ensured (see for instance the Ref. [4] with a discussion in this topic for the Horava–Lifshitz gravity in the holographic context) and naturally imposes an upper limit for the bulk viscous stress.

Additionally to the cosmological constant, the expansion of the Universe can be driven by scalar fields and the appropriate election of a potential function for the scalar field. Some of these scalar fields are candidates for Dark Energy (DE), such as quintessence and tachyonic fields, however, for these models the parameter ω of the equation state (EoS) $p = \omega\rho$ is restricted to $\omega > -1$. Nevertheless, more than a decade ago was pointed out in [5,6] that $\omega < -1$ is also compatible with most classical tests of cosmology, including the type 1a SNe data as well as the cosmic microwave background anisotropy and mass power spectrum. More recent observations do not ruled out this possibility [7–11]. In order to explore this new region it was necessary the introduction of a new component given by a scalar field called *phantom* with "wrong" sign in its kinetic term, $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$, such that $\rho + p = -\dot{\phi}^2 < 0$. This scenario is not consistent with the dominant energy component (DEC), so an increasing energy density for the fluid filling

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the Universe occurs as the cosmic time evolves. These phantom fields have the drawback of experimenting vacuum instability [12], so further investigations have been devoted to cross this phantom divide without instabilities. In the case of general k-essence models for a single scalar field which is described by an action with interaction with other energy components throughout kinetic couplings and higher derivatives, can not cross the phantom divide without gradient instabilities, singularities or ghosts. Another possibility, as suggested in [13], corresponds to the treatment of the scalar field as a velocity potential of an imperfect fluid, when the expansion around a perfect fluid is considered, the identification of some terms which correct the pressure in the manner of bulk viscosity can be performed.

In the case of imperfect fluids in the framework of General Relativity it is well known that the existence of dissipation in the cosmic components is a mechanism that allows violation of DEC condition [14]. Then, it is natural to explore the existence of phantom solutions in the causal thermodynamics approach of the IS theory. In this framework a phantom solution was recently found in Ref. [15]. In this solution the Universe is filled with a single fluid with positive pressure and the viscosity drives the effective EoS to be of phantom type. So this solution can be interpreted as an indication of the possibility to cross the phantom divide via a dissipative mechanism. Nevertheless, a phantom solution do not satisfy the near equilibrium condition, given by

$$\left|\frac{\Pi}{\rho}\right| << 1, \tag{1}$$

where Π is the viscous pressure and ρ the energy density of the fluid. This is because for an accelerated expansion the condition $\ddot{a} > 0$ leads to

$$-\Pi > p + \frac{\rho}{3}. \tag{2}$$

Then, the inequality given by the Eq. (2) implies that the viscous stress is greater than the equilibrium pressure p. The IS approach is valid in the near equilibrium regime, so, in order to include the phantom case it is necessary to consider a non-linear generalization of the causal linear thermodynamics of bulk viscosity where deviations from equilibrium are allowed. In this non-linear approach accelerated stable expansion has been reported for non-interacting two-fluid models where some of them presents viscosity [16,17]. In this work we focus on a non-linear extension of the IS formalism to explore the possibility of a phantom solution, and how it can be characterized in terms of the parameters associated to the dissipative fluid involved.

The organization of this article is as follows: In Section 2, by considering the non-linear extension of the IS theory made by Maartens and Méndez [3], we explore the existence of a phantom solution for the dynamical equation of the non-linear bulk viscosity. This is done by using the following Ansatz

$$H\left(t\right) = A\left(t_s - t\right)^{-1}, \tag{3}$$

which leads to a phantom solution for a late time FLRW flat universe filled with only one barotropic fluid with bulk viscosity, where A is a constant and t_s a finite time in the future [18]. In Section 3 we consider some thermodynamical implications of the phantom solution. In Section 4 we write some general comments and the conclusions of the work.

2. Phantom solution in the non-linear regime

For the obtention of a phantom solution in a non-linear extension of the causal IS formalism, we will follow the line of reasoning provided in Ref. [3] where the universe has an unique component given by a fluid described by its pressure and density, denoted by p and ρ respectively. When scalar dissipative effects are considered, the large deviations from equilibrium arise from large bulk viscous stresses, this can be translated as $|\Pi| \geq p$, where p is the local equilibrium pressure, and the non-equilibrium pressure is

$$p_{eff} = p + \Pi. \tag{4}$$

For a covariant approach of causal thermodynamics of relativistic fluids, we make use of some hydrodynamic tensors together with the standard definition of some cosmological quantities. We define n^α as the particle number four-current and the entropy four-current S^α. The non-negative entropy production rate or second law of thermodynamics $S^\alpha{}_{,\alpha} \geq 0$, must be guaranteed. If we discard vector and tensor dissipation, we can express the entropy four-current as

$$S^\alpha = S_{eff} n^\alpha, \tag{5}$$

where S_{eff} is the effective non-equilibrium specific entropy. In IS formalism, S_{eff} is given by the expression

$$S_{eff} = S - \left(\frac{\tau}{2nT\zeta}\right)\Pi^2, \tag{6}$$

where $\zeta(\rho, n)$ is the bulk viscosity, $\tau(\rho, n)$ is the characteristic time for linear relaxational or transient effects and T, n are the temperature and number density, respectively. S and T are local equilibrium variables which satisfy the Gibbs equation

$$TdS = (\rho + p)d\left(\frac{1}{n}\right) + \frac{1}{n}d\rho. \tag{7}$$

In the IS framework, the viscous pressure obeys the following transport equation

$$\tau\dot{\Pi} + \left(1 + \frac{1}{2}\tau\triangle\right)\Pi = -3\zeta H, \tag{8}$$

where we have defined

$$\triangle := 3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}. \tag{9}$$

On the other hand, the second law is satisfied by considering a linear relation between the thermodynamic "force" $\mathcal{X}$ and viscous pressure Π, i.e., $\Pi = -\zeta\mathcal{X}$ where

$$\mathcal{X} = 3H + \frac{\tau}{\zeta}\dot{\Pi} + \frac{\tau}{2\zeta}\Pi\,\triangle\,. \tag{10}$$

The non-linear effects can be introduced by considering the viscous pressure as [3]

$$\Pi = -\frac{\zeta\mathcal{X}}{1 + \tau_*\mathcal{X}}, \tag{11}$$

where $\tau_* \geq 0$ is the characteristic time for non-linear effects and is defined as

$$\tau_* = k^2\tau, \tag{12}$$

being k a constant. In the case $k = 0$ we obtain the linear IS theory. The linear relaxation time and the bulk viscosity are related as

$$\tau = \frac{\zeta}{v^2(\rho + p)}, \tag{13}$$

being v^2 the dissipative contribution to the speed of sound, V. The causality condition demands $V^2 = v^2 + c_s^2 \leq 1$, where $c_s^2 = (\partial p/\partial\rho)_S$ is the adiabatic contribution. In the barotropic case, $p = \omega\rho$, we obtain $c_s^2 = \omega$ then $v^2 \leq 1 - \omega$. In the non-linear case