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Pseudo Nambu–Goldstone modes in neutron stars

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ABSTRACT

If quarks and gluons are either gapped or confined in neutron stars (NSs), the most relevant light modes are Nambu–Goldstone (NG) modes. We study NG modes within a schematic quark model whose parameters *at high density* are constrained by the two-solar mass constraint. Our model has the color-flavor-locked phase at high density, with the effective couplings as strong as in hadron physics. We find that strong coupling effects make NG modes more massive than in weak coupling predictions, and would erase several phenomena caused by the stressed pairings in mismatched Fermi surfaces. For instance, we found that charged kaons, which are dominated by diquark and anti-diquark components, are not light enough to condense at strong coupling. Implications for gravitational wave signals for NS–NS mergers are also briefly discussed.

1. Introduction

Neutron stars are unique cosmic laboratories to study cold dense QCD. The observations of two-solar mass ($2M_\odot$) neutron stars (NSs) [1,2] tell us that equations of state at high baryon density, $n_B \gtrsim 5n_0$ ($n_0 \simeq 0.16\,\mathrm{fm}^{-3}$: nuclear saturation density), must be very stiff to prevent the star from gravitational collapsing. Meanwhile low density equations of state at $n_B \lesssim 2n_0$ should be softer than the previous thoughts, as suggested by studies of neutron star radii [3–7], nuclear symmetry energy, heavy ion data [8], and predictions of chiral effective theories [9] that are combined with sophisticated many-body calculations [10,11]. Then the theoretical challenge is how to reconcile these tendencies at low and high density [12,13]; to connect soft and stiff equations of state, there must be a region where the sound velocity $c_s = (\partial P/\partial \varepsilon)^{1/2}$ is large, but this tends to violate the causality constraint $c_s \le 1$. Also, soft-to-stiff equations of state does not allow us to implement strong first order phase transitions which soften equations of state at high density.[1] In this way the constraints from high and low density *together* limit the possible classes of equations of state, and from which one can extract useful insights into matter [14].

While the current observations have already provided significant constraints, equations of state alone do not finalize our understanding of dense matter. It is desirable to study dynamic and thermal aspects of matter which originate from excitation modes whose properties are very sensitive to the phase structure. The predictions for thermal equations of state are very important for the gravitational wave astronomy [16,17]; the first direct detection of gravitational waves of a binary black-holes has been made in September 2015 [18], and the second detection three months later [19]. We also expect gravitational waves from NS–NS mergers in near future. It has been argued that the patterns of gravitational waves can discriminate different equations of state by comparing the observations with the waveforms predicted by numerical relativity with input equations of state [20].

The purpose of this paper is to study the low energy modes at high density, in a setup consistent with neutron star constraints at zero temperature. Our underlying physical picture is based on a 3-window description[2] for dense QCD matter which has been studied in recent works [12,22–24]. The 3-window picture consists of: purely nuclear matter for $n_B \lesssim 2n_0$; quark matter for $n_B \gtrsim 5n_0$; and crossover (or weak 1st order) picture[3] for $2n_0 \lesssim n_B \lesssim 5n_0$. Each domain has own characteristic constraint; nuclear physics constraint for $n_B \lesssim 2n_0$; the $2M_\odot$ constraint for $n_B \gtrsim 5n_0$; and the causality and thermodynamic constraints on $2n_0 \lesssim n_B \lesssim 5n_0$ which require $0 \le c_s^2 \le 1$. To express the nuclear constraints for

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[1] If we assume very stiff hadronic equations of state at low density, we may allow strong first order phase transitions from very stiff hadronic matter to quark matter stiff enough to pass the $2M_\odot$ constraint [15].

[2] Somewhat indirect, but less model-dependent approach is to interpolate the nuclear and perturbative quark matter equations of state. This has been carried out with perturbative calculations to three loop order [21].

[3] The crossover picture has been also discussed in an effective Lagrangian framework in [25], where matter changes its character but does not induce the first order phase transition.

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$n_B \lesssim 2n_0$, in [12] we used the Akmal–Pandharipande–Ravenhall equation of state [26] as a representative. Three descriptions are matched smoothly at the boundaries.

Our quark matter description is based on a schematic quark model of the Nambu–Jona-Lasinio (NJL) type with supplemental effective vector and color-magnetic interactions. The additional interactions are included to mimic important aspects of nuclear physics and hadron spectra, and this is consistent with the crossover picture at intermediate density. In fact these interactions have played very important roles in constructing physically sensible equations of state in our 3-window treatment [12]. The parameters should be regarded as those at $n_B \gtrsim 5n_0$; in general effective couplings can be medium dependent. The running vector coupling was manifestly taken into account in Ref. [27] as an illustration. In the previous studies we found that those couplings should be as large as the vacuum coupling constants, otherwise quark model equations of state easily show strong first order phase transitions to become the ideal gas equation of state, which is too soft.

We construct NG modes in this strong coupling setup. In the present model quark matter at $n_B \gtrsim 5n_0$ appears to be the color-flavor-locked (CFL) phase [28]. The studies of NG modes in the CFL phase are not quite new; in fact there are several elegant studies based on weak coupling pictures and effective Lagrangian approach which provided us with analytic insights [29–33]. Model studies have been done in [35–37].

Nevertheless we feel it necessary to update model analyses for several reasons: (i) for the construction of thermal equations of state, we need to discuss quantitative aspects of NG modes in a consistent way with the neutron equations of state at zero temperature; (ii) the strong couplings in our setup can alter weak coupling predictions at qualitative level. In particular the strong pairing effects overcome the stress in mismatched Fermi surfaces, erasing several instabilities; (iii) in the previous model studies at strong coupling, the coexistence of chiral and diquark condensates is not realized because they are separated by the first order phase transition. But if there exists a mechanism tempering the growth of quark number density, we readily find the coexistence region [38]. In our modeling with vector interactions, the considerable amount of chiral condensates can remain at $n_B \gtrsim 5n_0$. So it is important to clarify how the residual chiral condensates affect the spectra of NG modes; (iv) the NG modes in the coexistence of chiral and diquark condensates are discussed by effective Lagrangian approach in [39]. But this study is limited to the 3-flavor limit, and we need to add charge neutrality constraints and explicit flavor symmetry breaking for the application to neutron star physics; (v) in our strong coupling picture the gluons remain non-perturbative to $n_B \sim 10n_0$, so we should also keep the $U_A(1)$ breaking for consistency. The quark matter with non-perturbative gluons has been addressed in the context of quarkyonic matter [40].

In this work we will study the NG modes which acquire masses and effective chemical potentials through the explicit flavor symmetry breaking. They are NG modes associated with the breaking of approximate chiral symmetry and color symmetry, $SU(3)_L \times SU(3)_R \times SU(3)_c \to SU(3)_{c+L+R}$. If the $U_A(1)$ breaking by quantum effects appears to be negligible, there is an additional NG mode. As a whole we have $8+1$ NG modes which can be labelled by the same quantum numbers as in the vacuum, (π, K, η, η'). We will study these modes within the random phase approximation (RPA). In this study we will not discuss the NG mode associated with the $U(1)_B$ breaking which is known to be strictly massless.

2. A schematic quark model

We introduce our model Lagrangian for a 3-flavor quark matter. Our notation is as follows: the metric is $g_{\mu\nu} = \mathrm{diag.}(1, -1, -1, -1)$; the flavor matrices consist of $\tau_0 = \sqrt{2/N_\mathrm{f}}$ and the Gell-Mann matrices $\tau_a (a = 1, \cdots, 8)$; the color matrices are the Gell-Mann matrices $\lambda_a (a = 1, \cdots, 8)$; the charge matrix is $\hat{Q} = \mathrm{diag.}(-2/3, 1/3, 1/3)$.

Our model Lagrangian for quarks and leptons, including the chemical potentials, is

$$\mathcal{L} = \bar{q}\left(\mathrm{i}\partial\!\!\!/ - \hat{m} + \hat{\mu}\gamma_0\right) q + \sum_{i=e,\mu} \bar{l}_i \left(\mathrm{i}\partial\!\!\!/ - m_i - \mu_Q \gamma_0\right) l_i + \mathcal{L}_{\mathrm{NJL}} + \mathcal{L}_{\mathrm{mag}} + \mathcal{L}_{\mathrm{BB}}\,, \tag{1}$$

where $\hat{m} = \mathrm{diag.}(m_u, m_d, m_s)$, m_e and m_μ are masses for electrons and muons, and the chemical potentials for quarks are

$$\hat{\mu} = \mu_q + \mu_3^c \lambda_3 + \mu_8^c \lambda_8 + \mu_Q \hat{Q}\,. \tag{2}$$

The thermodynamic variable is μ_q while the electric charge chemical potential, μ_Q, and the color chemical potentials, μ_3^c, μ_8^c, are used as the Lagrange multipliers to guarantee the charge and color neutrality constraints, respectively.

There are several important interactions for our studies of neutron stars and the QCD phase diagram. $\mathcal{L}_{\mathrm{NJL}}$ is the standard NJL interaction including the chiral 4-Fermi and Kobayashi–Maskawa–'tHooft (KMT) interactions,

$$\mathcal{L}_{\mathrm{NJL}} = \frac{G_s}{2} \sum_{a=0}^{8} \left[(\bar{q}\lambda_a q)^2 + (\bar{q}\lambda_a \mathrm{i}\gamma_5 q)^2 \right] - K \left(\det_{\mathrm{f}}[\bar{q}(1-\gamma_5)q + \det_{\mathrm{f}}[\bar{q}(1+\gamma_5)q] \right)\,. \tag{3}$$

In addition to this, we add effective interaction, $\mathcal{L}_{\mathrm{mag}}$, inspired from the color-magnetic interaction which plays important roles to describe the pattern of the hadron spectra in quark models. It is given by

$$\mathcal{L}_{\mathrm{mag}} = \frac{H}{2} \sum_{A,A'=2,5,7} \left[(\bar{q}\,\mathrm{i}\gamma_5 \tau_A \lambda_{A'} q_C)(\bar{q}_C\,\mathrm{i}\gamma_5 \tau_A \lambda_{A'} q) + (\bar{q}\tau_A \lambda_{A'} q_C)(\bar{q}_C \tau_A \lambda_{A'} q) \right], \tag{4}$$

where the specific choice of the matrices $A, A' = 2, 5, 7$ makes the attractive s-wave interactions anti-symmetric in color and flavor. Finally we consider the repulsive vector interaction,

$$\mathcal{L}_{\mathrm{BB}} = -\frac{G_V}{2} (\bar{q}\gamma_\mu q)^2\,, \tag{5}$$

which is inspired from the ω-meson exchange between nucleons.

The model will be treated within the mean field approximation. The mean fields for the chiral condensates are $\sigma_i = \langle \bar{q}_i q_i \rangle$; quark number density $n = \sum_{i=u,d,s} \langle q_i^\dagger q_i \rangle$; and diquark condensates $d_i = \langle q^T C \mathrm{i}\gamma_5 R_i q \rangle$ where $(R_u, R_d, R_s) \equiv (\tau_7\lambda_7, \tau_5\lambda_5, \tau_2\lambda_2)$. With these definitions, the diquark condensates (d_u, d_d, d_s) correspond to (ds, su, ud) quark pairings, respectively.

The mean field particle propagators contain these mean field effects; the inverse of the propagator $S(k)$ is given by

$$S^{-1}(k) = \begin{pmatrix} k\!\!\!/ - \hat{M} + \hat{\mu}\gamma^0 & \gamma_5 \Delta_i R_i \\ -\gamma_5 \Delta_i^* R_i & k\!\!\!/ - \hat{M} - \hat{\mu}\gamma^0 \end{pmatrix}, \tag{6}$$

where the effective Dirac and Majorana masses are given by

$$M_i = m_i - 2G_s \sigma_i + K|\epsilon_{ijk}|\sigma_j \sigma_k\,, \qquad \Delta_i = -H d_i\,. \tag{7}$$

The thermodynamic potential for the quark part now reads