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Topological charged black holes in massive gravity's rainbow and their thermodynamical analysis through various approaches

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ABSTRACT

Violation of Lorentz invariancy in the high energy quantum gravity motivates one to consider an energy dependent spacetime with massive deformation of standard general relativity. In this paper, we take into account an energy dependent metric in the context of a massive gravity model to obtain exact solutions. We investigate the geometry of black hole solutions and also calculate the conserved and thermodynamic quantities, which are fully reproduced by the analysis performed with the standard techniques. After examine the validity of the first law of thermodynamics, we conduct a study regarding the effects of different parameters on thermal stability of the solutions. In addition, we employ the relation between cosmological constant and thermodynamical pressure to study the possibility of phase transition. Interestingly, we will show that for the specific configuration considered in this paper, van der Waals like behavior is observed for different topology. In other words, for flat and hyperbolic horizons, similar to spherical horizon, a second order phase transition and van der Waals like behavior are observed. Furthermore, we use geometrical method to construct phase space and study phase transition and bound points for these black holes. Finally, we obtain critical values in extended phase space through the use of a new method.

1. Introduction

It is arguable that Einstein gravity is an effective theory which is valid in infrared (IR) limit while in ultraviolet (UV) regime, it fails to produce accurate results. This shortcoming requires modification in order to incorporate the UV regime. It is believed that the Lorentz symmetry is an effective symmetry in IR limit of quantum gravitational processes. Since the standard energy–momentum dispersion relation depends on such symmetry, one expects to regard the modified energy–momentum dispersion relation in UV regime. Such modification motivates one to develop double special relativity [1,2], in which this theory has two upper bounds (speed of light (c) and the Planck energy (E_P)) [1]. In this theory, it is not possible for a particle to achieve velocity and energy larger than the speed of light and the Planck energy, respectively [3]. Generalization of this doubly special relativity to curved spacetime is gravity's rainbow [3]. On the other hand, if one considers the gravity as an emerging phenomenon due to quantum degrees of freedom, spacetime should be described with an energy dependent metric. Hence, the spacetime is affected by a particle probing it and since this particle can acquire a range of energies (E), a rainbow of energy is built. The gravity's rainbow can be constructed through the use of deforming the standard energy–momentum relation as $E^2 f^2(\varepsilon) - P^2 g^2(\varepsilon) = m^2$, in which $\varepsilon = E/E_P$, E is the energy of that test particle probing the geometry of spacetime and the functions $f^2(\varepsilon)$ and $g^2(\varepsilon)$ are called rainbow functions. It is notable that, in order to recover the standard energy–momentum relation in the IR limit, the rainbow functions satisfy the following relation

$$\lim_{\varepsilon\to 0} f(\varepsilon) = \lim_{\varepsilon\to 0} g(\varepsilon) = 1. \tag{1}$$

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Now, it is possible to define an energy dependent deformation of the metric $\hat{\mathbf{g}}$ with the following form [3]

$$\hat{\mathbf{g}} = \eta^{\mu\nu} e_\mu(E) \otimes e_\nu(E), \tag{2}$$

where $e_0(E) = \frac{1}{f(\varepsilon)}\hat{e}_0$ and $e_i(E) = \frac{1}{g(\varepsilon)}\hat{e}_i$, in which the hatted quantities refer to the energy independent frame.

On the other hand, gravity's rainbow has specific properties which were highlighted in recent studies. At first, we point out that this theory enjoys a modification in energy–momentum dispersion relation. Such modification in the UV limit is examined in studies that were conducted in discrete spacetime [4], models based on string theory [5], spacetime foam [6], spin-network in loop quantum gravity (LQG) [7], non-commutative geometry [8], Horava–Lifshitz gravity [9,10] and also ghost condensation [11]. In addition, the observational evidences confirm that such modification could exist [12]. Second, it was pointed out that by treatment of the horizon radius of black holes as radial coordinate in this theory, the usual uncertainty principle stands [13,14]. It is possible to translate the uncertainty principle ($\Delta p \geq 1/\Delta x$) into a bound on the energy ($E \geq 1/\Delta x$) where E can be interpreted as the energy of a particle emitted in the Hawking radiation process. Also, it has been shown that the uncertainty in the position of a test particle in the vicinity of horizon should be equal to the event horizon radius (see Refs. [15–18] for more details) as $E \geq 1/\Delta x \approx 1/r_+$, in which E is the energy of a particle near the horizon that is bounded by the E_P and cannot increase to arbitrary values. Hence, this bound on the energy modifies the temperature and the entropy of black holes in gravity's rainbow [18]. Third, it was shown that the black hole thermodynamics in the presence of gravity's rainbow is modified. Such modification leads to results such as existence of remnant for black holes [18,19] which is proposed to be a solution to information paradox [20].

Recently, the theoretical aspects of gravity's rainbow have been investigated in various contexts [21–29]; different classes of black holes with different gauge fields have been studied in Refs. [18,30–32]. In addition, the hydrostatic equilibrium equation of stars and the effects of this generalization on neutron stars have been examined in Ref. [33]. Also, wormhole solutions in gravity's rainbow were obtained in Ref. [34]. Besides, the effects of rainbow functions on gravitational force are studied in Ref. [35].

The fundamental motivation of considering gravity's rainbow comes from the violation of Lorentz invariancy (or diffeomorphism invariancy) in the high energy regime. In this regime, one may regard a massive deformation of standard general relativity to obtain a Lorentz-invariant theory of massive gravity as well. It was shown that the Lorentz-breaking mass term of graviton leads to a physically viable ghost free model of gravity [36]. Although the graviton in general relativity is considered as a massless particle, there are some arguments regarding the existence of massive gravitons. The first attempt for building such theory was done by Fierz–Pauli [37] which Boulware and Deser have shown that it suffers the ghost instability in nonlinear extension [38]. There have been several reports regarding the interaction effects of nonlinear theory of massive gravity in the absence of ghost field [39–42]. Charged black holes in the presence of massive gravity were investigated in Refs. [43,44] (see [45–49] for more details regarding considering massive gravity). Moreover, phase transition and entanglement entropy of a specific massive theory were studied in Ref. [50]. On the other hand, de Rham, Gabadadze and Tolley (dRGT) proposed another theory of massive gravity [51] without Boulware–Deser ghost [52–54]. dRGT massive gravity employs a reference metric for constructing mass terms. After that, Vegh used a singular metric for constructing a dRGT like theory [55]. In his theory, the graviton behaves like a lattice in specific limits and a Drude peak was observed. It was shown that for an arbitrary singular metric the mentioned theory is ghost free [56] and enjoys stability which was addressed in Refs. [53,54]. Different classes of exact black hole solutions in the presence of this massive gravity and their thermodynamics, phase transition, geometrical thermodynamics and their thermal stability have been investigated [57–59]. In addition, the holographic superconductor–normal metal–superconductor Josephson junction for this specific massive gravity has been studied and it was shown that massive gravity has specific contributions to its properties [60].

It has been proposed that a consistent quantum theory of the gravity may be obtained through the use of black holes thermodynamics. The interpretation of geometrical aspect of black holes as thermodynamical variable provides a powerful viewpoint for constructing such theory. In addition, the recent advances in gauge/gravity duality emphasize on importance of black holes thermodynamics [61–74]. On the other hand, the pioneering work of Hawking and Page, which was based on the phase transition of asymptotically adS black holes [75], and also Witten's paper on the similar subject [76] highlighted the importance of black holes thermodynamics. In order to study black hole thermodynamics, we can use different approaches which are based on various ensembles. One of the thermodynamical aspects of black holes is investigation of thermal stability in the canonical ensemble. In the canonical ensemble, the sign of heat capacity determines thermal stability/instability of the black holes. In addition, the roots and divergencies of heat capacity are denoted as bound and phase transition point, respectively. Due to these reasons, black holes thermodynamics and their thermal stability have been investigated in literature [77–82].

Besides, the interpretation of cosmological constant as a thermodynamical variable (pressure) has been recently employed in literatures. Such consideration will lead to enriching the thermodynamical behavior of black holes and observation of specific properties of usual thermodynamical systems such as van der Waals like behavior [83–95], reentrant of phase transition [96] and existence of triple point for black holes [97]. In addition, the consequences of adS/CFT correspondence [79,98–100], ensemble dependency of BTZ black holes [101] and two dimensional dilaton gravity [102] justify the consideration of cosmological constant not as a fixed parameter but as a thermodynamical variable. Phase transition of different classes of black holes in the presence of different gauge fields has been investigated by employing such proportionality between the cosmological constant and thermodynamical pressure [103–108]. In order to study phase diagrams for obtaining critical values, in Ref. [109] a new method for calculating these critical values was introduced. In this method, by using a relation between the cosmological constant and pressure with denominator of the heat capacity of black holes, one can obtain a relation for the pressure. The maximum of pressure in this method is critical pressure in which the phase transition takes place. Such method has been employed in several papers [58,59,109] and it was shown to be a successful approach toward calculating the critical pressure and volume of black holes in extended phase space.

Another approach for studying the thermodynamical phase transition of black holes is through the use of a geometrical way. In this method, one can use thermodynamical variables of the black holes to construct a thermodynamical metric. The information regarding bound and phase transition point of the corresponding black hole is within the behavior of Ricci scalar of the constructed thermodynamical metric. In other words, bound and phase transition point of black holes are presented as divergencies in the mentioned Ricci scalar.

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