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Dissipationless Hall current in dense quark matter in a magnetic field

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ABSTRACT

We show the realization of axion electrodynamics within the Dual Chiral Density Wave phase of dense quark matter in the presence of a magnetic field. The system exhibits an anomalous dissipationless Hall current perpendicular to the magnetic field and an anomalous electric charge density. Connection to topological insulators and 3D optical lattices, as well as possible implications for heavy-ion collisions and neutron stars are outlined.

1. Introduction

In the past few years new macroscopically observable quantum effects that manifest through the interaction of matter with electromagnetic fields in QCD and condensed matter are attracting much attention [1,2]. These effects are connected to the nontrivial topology of these systems and are related to parity and charge-parity symmetry violations. The interaction between the electromagnetic field and matter with nontrivial topology is described by the equations of axion electrodynamics,

$$\nabla \cdot \mathbf{E} = J_0 + \kappa \nabla\theta \cdot \mathbf{B}, \quad (1)$$

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}_V - \kappa \left(\frac{\partial\theta}{dt}\mathbf{B} + \nabla\theta \times \mathbf{E}\right), \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad (3)$$

proposed by Wilzcek many years ago [3] to describe the effects of adding an axion term $\frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}$ to the ordinary Maxwell Lagrangian. In condensed matter, a term of this form has been shown to emerge in: 1) topological insulators (TI) [4], where θ depends on the band structure of the insulator, 2) Dirac semimetals (DM) [5], a 3D bulk analogue of graphene with non-trivial topological structures, and 3) Weyl semimetals [6], where the angle θ is related to the energy or momentum separation between the Weyl nodes.

For quark matter, an electromagnetic axion term can be generated via two separate mechanisms, one at high temperature (T) and the other at high density. At high T, a nontrivial axion field θ can arise thanks to the sphaleron transitions over the barrier that separates topologically inequivalent vacua [7]. Even though θ originally enters coupled to the gluon field, a Fujikawa transformation [8] eliminates such a term, but leads to the reappearance of θ in the QED sector of the theory, where it couples to the electromagnetic field $F_{\mu\nu}$ and its dual [2]. In the presence of a background magnetic field, the induced $\frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}$ term leads to electric charge separation through the well-known Chiral Magnetic Effect (CME) [9]. The mechanism at high density, takes place in the dual chiral density wave (DCDW) phase of dense quark matter in the presence of a magnetic field. The main purpose of this paper is to demonstrate the realization of this new mechanism and to show how it leads to the generation of a dissipationless Hall current and an anomalous electric charge.

We highlight from the onset that the role of the magnetic field is quite different at high T than at high density. While at high T the magnetic field just serves, via the CME, as a probe of the nontrivial QCD vacuum topology that produces the axion field θ; in the high-density case, the magnetic field is itself essential to produce the nontrivial topology that manifests, as it will become clear below, through the spectral asymmetry of the quarks in the lowest Landau level (LLL).

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2. Dissipationless Hall current in cold-dense quark matter at $B \neq 0$

Henceforth, we focus on the cold and dense region of QCD. In this region the CME should be suppressed, an observation consistent with the Beam Energy Scan (BES) results [10], which show that charge separation starts to diminish already at energies below 60 GeV and disappears completely between 19.6 and 7.7 GeV. A growing body of works indicates that with increasing density, the chirally broken phase of quark matter is not necessarily replaced by an homogeneous, chirally restored phase, but instead, at least for an intermediate region of densities, the system may favor the formation of inhomogeneous phases. To gain insight on why this occurs notice that with increasing density the homogeneous chiral condensate becomes disfavored due to the high-energy cost of exciting the antiquarks from the Dirac sea to the Fermi surface where the pairs form. At the same time, with higher densities, co-moving quarks and holes at the Fermi surface may pair with minimal energy cost through a mechanism analogous to Overhauser's [11], giving rise to a spatially modulated chiral condensate [12]. Spatially modulated chiral condensates in QCD have been discussed in the context of quarkyonic matter [13], where they appear in the form of quarkyonic chiral spirals [14] at zero magnetic field, or double quarkyonic chiral spirals [15] in the presence of a magnetic field. Inhomogeneous chiral condensates have been also studied in the context of NJL models (for a review see [16]) that share the chiral symmetries of QCD and are then useful to study the chiral phase transition.

From now on, we model cold and dense quark matter in a magnetic field with the help of the NJL-QED Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}\boldsymbol{\tau}\gamma_5\psi)^2], \tag{4}$$

with $Q = (e_u, e_d) = (\frac{2}{3}e, -\frac{1}{3}e)$, $\psi^T = (u, d)$; μ the baryon chemical potential; and G the four-fermion coupling. The electromagnetic potential A_μ is formed by the background $\bar{A}_\mu = (0, 0, Bx, 0)$, that corresponds to a constant and uniform magnetic field B in the z direction, plus the fluctuation field. The presence of the field B favors the formation of the DCDW condensate, $\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = \Delta e^{iqz}$ [17,18] with magnitude Δ and modulation vector $\mathbf{q} = (0, 0, q)$ along the field direction. In this phase, the mean-field Lagrangian is

$$\mathcal{L}_{MF} = \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu]\psi - m\bar{\psi}e^{i\tau_3\gamma_5 qz}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{4G}, \tag{5}$$

with $m = -2G\Delta$. The z-dependent mass term can be eliminated with the help of the local chiral transformation $\psi \to U_A\psi$, $\bar{\psi} \to \bar{\psi}\bar{U}_A$, with $U_A = e^{-i\tau_3\gamma_5\theta}$, $\bar{U}_A = \gamma_0 U^{\dagger}\gamma_0 = e^{-i\tau_3\gamma_5\theta}$, and $\theta(t, \mathbf{x}) = \frac{1}{2}q_\mu x^\mu = qz/2$, so that now

$$\mathcal{L}_{MF} = \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu} - i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{4G} \tag{6}$$

The energy spectrum of the quarks in (6) separates into the LLL ($l = 0$)

$$E_k^{LLL} = \epsilon\sqrt{\Delta^2 + k_3^2} + q/2, \quad \epsilon = \pm, \tag{7}$$

and the higher ($l > 0$) Landau level

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, \ldots \tag{8}$$

modes. Notice that the LLL spectrum is not symmetric about the zero energy level [17,18]. The asymmetry of the spectrum is characterized by a topological quantity, known as the Atiyah–Patodi–Singer invariant $\eta_B = \sum_k \mathrm{sgn}(E_k)$ [19], a quantity related to the chiral anomaly [18]. This sum is divergent and needs to be properly regularized to ensure that all the energies with equal magnitude and opposite signs cancel out. This implies that only the asymmetric modes contribute to η_B and hence the anomalous effects of the system are connected to the LLL. The regularized index $\eta_B = \lim_{s\to 0}\sum_k \mathrm{sgn}(E_k)|E_k|^{-s}$ gives rise to an anomalous baryon (quark) number density ρ_B^A. Regularizing the sum with the help of a Mellin transform [18] leads to

$$\rho_B^A = -N_c\eta_B/2 = N_c\sum_f \frac{|e_f|}{4\pi^2}\mathbf{B}\cdot\nabla(\mathbf{q}\cdot\mathbf{x}) = 3\frac{|e|}{4\pi^2}qB \tag{9}$$

for the case $q < 2m$. The use of a different regularization procedure that allows to extract the anomalous part of the thermodynamic potential, led to the same anomalous quark number density, obtained in this case not from the index η_B, but as the derivative of the thermodynamic potential with respect to the baryon chemical potential μ [17]. The extension of this calculation to the isospin asymmetric case was done in [20]. When $q > 2m$, the quark density acquires an additional, non-topological contribution [18] and becomes

$$\rho_B^A = -N_c\eta_B/2 = -N_c|eB|\left[-q + \sqrt{q^2 - 4m^2}\right]/4\pi^2 \tag{10}$$

Notice that if $B = 0$, the quark spectrum is symmetric, so η_B vanishes and no anomalous quark number density exists. On the other hand, if $B \neq 0$, but $m = 0$, we have $\rho_B^A = 0$, consistent with the fact that in this case there is no DCDW condensate, therefore no dependence on the modulation parameter q must remain and no anomalous baryon charge must exist. These considerations underline that the nontrivial topology of this model results from the interplay of the DCDW ground state and the magnetic field.

At this point, we should notice that the fermion measure in the path integral is not invariant under the U_A transformation that led to (6), because $\bar{U}_A = U_A \neq U_A^{-1}$ and thus $D\bar{\psi}D\psi \to (\det U_A)^{-2}D\bar{\psi}D\psi$. Hence, it follows that $(\det U_A)^{-2} = e^{-2\mathrm{Tr}\log U} = e^{-2i\int d^4x\theta(x)\delta^{(4)}(0)\mathrm{tr}\tau_3\gamma_5}$, with Tr a functional and matrix trace, and tr a matrix trace. This expression is ill defined and needs a gauge-invariant regularization. With that aim, we consider a smooth function $f(t)$, such that $f(0) = 1$, $f(\infty) = 0$, and $tf'(t) = 0$ at $t = 0$ and $t = \infty$, and regularize the exponent as

$$\int d^4x\theta(x)(-2\delta^{(4)}(0)\mathrm{tr}\tau_3\gamma_5) = -2\lim_{\Lambda\to\infty}\mathrm{Tr}\left[\theta(x)\tau_3\gamma_5 f((iD_\mu\gamma^\mu/\Lambda)^2)\right], \tag{11}$$

where $D_\mu = \partial_\mu + iQA_\mu - i\tau_3\gamma_5\frac{q}{2}$ is the corresponding Dirac operator. Following Fujikawa's approach [8], one can show that $(\det U_A)_R^{-2} = e^{i\int d^4x\frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}}$. The correct effective Lagrangian is then

$$\mathcal{L}_{eff} = \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu} - i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{m^2}{4G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}, \tag{12}$$