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Resonance and continuum Gamow shell model with realistic nuclear forces

Z.H. Sun, Q. Wu, Z.H. Zhao, B.S. Hu, S.J. Dai, F.R. Xu *

School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

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ABSTRACT

Starting from realistic nuclear forces, we have developed a core Gamow shell model which can describe resonance and continuum properties of loosely-bound or unbound nuclear systems. To describe properly resonance and continuum, the Berggren representation has been employed, which treats bound, resonant and continuum states on equal footing in a complex-momentum (complex-k) plane. To derive the model-space effective interaction based on realistic forces, the full $\hat{Q}$-box folded-diagram renormalization has been, for the first time, extended to the nondegenerate complex-k space. The CD-Bonn potential is softened by using the $V_{\text{low-}k}$ method. Choosing ^{16}O as the inert core, we have calculated sd-shell neutron-rich oxygen isotopes, giving good descriptions of both bound and resonant states. The isotopes ^{25,26}O are calculated to be resonant even in their ground states.

Resonance is a general phenomenon happening in classic or quantum systems. It plays a special role in weakly-bound or unbound quantum systems. An unbound quantum system, such as atomic cluster or unbound nucleus, can emerge in the form of intrinsic resonance. Nuclear resonances are usually followed by particle emissions as the Gamow quantum tunneling. Weakly-bound or unbound nuclei are complex open quantum systems (OQS) in which the coupling to the scattering continuum is crucial and should be properly treated. A small uncertainty in modeling would change the conclusions of physics.

In the standard shell model (SM), harmonic oscillator (HO) wave functions are always bound and localized, while a loosely-bound nucleus has small separation energy and large spatial spread, in which the continuum plays a critical role. To overcome the shortcoming of the conventional SM, the continuum shell model (CSM) [1–3] has been developed, taking into account the continuum effect by projecting the model space onto the subspaces of bound and scattering states in a real-energy basis. The continuum effect has also been well treated in the continuum coupled cluster [4] and the continuum-coupled shell model [5].

The Gamow resonance is in fact a time-dependent problem, associated with a decaying process. However, the exact treatment of the time-dependent problem is difficult. Berggren generalized the time-independent Schrödinger equation to a complex-k plane, giving single-particle (SP) bound states, unbound resonant states and nonresonant continuum states [6]. The three types of the SP states construct a complete set of basis states, called the Berggren ensemble [6]. Using the Berggren basis, the so-called Gamow shell model (GSM) has been advanced recently. With phenomenological interactions, the GSM has been successfully applied to nuclear structure calculations [7–10].

It is pursued currently to perform the first-principles calculations of nuclear structure. Such calculations require two fundamental elements: (i) using realistic nuclear forces and (ii) rigorously treating many-body correlations. The shell model which is beyond mean-field approach provides a good platform to handle many-body correlations. Starting from realistic forces, traditional core shell model [11,12] or no core shell model (NCSM) calculations [13] have obtained great success, while the GSM calculation faces big challenges. For example, the wave functions of resonant SP states are not square integrable, which complicates severely the treatments of the complex non-Hermitian Hamiltonian and other mechanical quantities. It is even more challenging to establish a GSM based on realistic nuclear forces with an inert core. In this case, one has to strive to build the model-space effective interaction in the complex-k basis. Nevertheless, the realistic core GSM (CGSM) calculation has been proposed with limit to two- or three-particle systems [14,15]. The effective interaction was built by using the degenerate $\hat{Q}$-box approach but neglecting folded diagrams

* Corresponding author.
E-mail address: frxu@pku.edu.cn (F.R. Xu).

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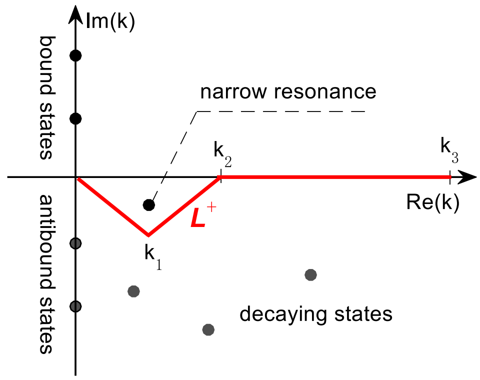

Fig. 1. Schematic Berggren complex-k plane. The scattering states lie on the L^+ contour (red line). The bound, resonant and scattering states construct the Berggren completeness relation. The contour L^+ has to be chosen in such a way that all the discrete narrow resonant states are contained in the domain between L^+ and the real k axis. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

[15]. The folded-diagram procedure is to sum up the subsets of diagrams to infinite order [12]. Recently, realistic no-core Gamow shell model (NCGSM) was developed for the light nuclei of helium isotopes [16], in which one escapes the task of building effective interaction.

In the present work, we start from realistic nuclear forces to develop a sophisticated CGSM for many-particle nuclear systems. The inert core takes a doubly magic nucleus. The realistic effective interaction in the model space is obtained by extending the non-degenerate $\hat{Q}$-box folded-diagram renormalization [17,18] to the complex-k space. With choosing ^{16}O as the inert core, we have calculated the sd-shell oxygen isotopes up to the unbound ^{26}O nucleus.

The Berggren basis is generated by the Woods–Saxon (WS) potential including spin-orbit coupling [19]. The radial wave functions of the Berggren SP states are obtained by solving the time-independent Schrödinger equation in the complex-k space,

$$\frac{d^2u(k,r)}{dr^2} = \left[\frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}U(r) - k^2\right]u(k,r), \tag{1}$$

where l is the orbital angular momentum of the nucleon motion. The momentum k and wave function $u(k,r)$ can be complex numbers (or function). $U(r)$ is the spherical WS potential generated by the core. There is no Coulomb interaction for neutrons. The piecewise perturbation method [20] has been used to solve the Berggren SP eigen equation (1). For a resonant state, the eigen energy is a complex number, $\tilde{e}_n = e_n - i\gamma_n/2$, where γ_n stands for the resonance width.

Fig. 1 shows a schematic Berggren complex-k plane. The Berggren SP states form a complete set of basis states with discrete bound states, resonant states and continuum scattering states [6]. The wave functions of resonant states are not square integrable. The exponential increase and infinitely oscillating behavior make that the resonant wave function cannot be normalized with the conventional techniques. In the present work, we use the exterior complex scaling method [21] for the normalizations of resonant states.

We use the universal WS parameters [19], but reduce the strength $|V_0|$ by 2.3 MeV to obtain a reasonable $0d_{3/2}$ resonance width compared with the experimental width extracted in ^{17}O [9]. The WS potential of the ^{16}O core gives two bound orbits $0d_{5/2}$ and $1s_{1/2}$ at energies -5.31 and -3.22 MeV, respectively, and a resonant orbit $0d_{3/2}$ with energy $\tilde{e} = 1.06 - 0.09i$ MeV. The higher orbits $f_{7/2,5/2}$, $np_{3/2,1/2}$ $(n \geq 1)$, $g_{9/2,7/2}$, $nd_{5/2,3/2}$ $(n \geq 1)$ and $ns_{1/2}$ $(n \geq 2)$ are continuum partial waves. We take the $l \leq 4$ orbits for the sd-shell calculations. The model space for the GSM calculation is $\{1s_{1/2}, 0d_{5/2}, 0d_{3/2} + d_{3/2}\text{-continuum}\}$. $0d_{3/2}$ is a narrow resonance state which plays a crucial role in the descriptions of the sd-shell nuclear resonances. In this case, the coupling between the $0d_{3/2}$ resonance and $d_{3/2}$ continuum needs to be treated carefully. Therefore, the $d_{3/2}$ continuum is included as the part of the model space in the GSM calculation. Contributions from the $0s_{1/2}0p_{3/2,1/2}$ core polarization and other continuum partial waves are taken into account by performing the $\hat{Q}$-box folded-diagram calculations. In practical computations, the continuum states on the contour L^+ need to be discretized. We use the Gauss–Legendre quadrature method [10,14,22] for the discretization.

The intrinsic A-body Hamiltonian has the following form,

$$H = \sum_{i=1}^{A}\frac{p_i^2}{2m} + \sum_{i<j}^{A} v_{ij} - \frac{\mathbf{P}^2}{2Am}, \tag{2}$$

where v_{ij} is the nucleon–nucleon interaction. p_i is the nucleon momentum in the laboratory coordinate, while $\mathbf{P} = \sum_{i=1}^{A}\mathbf{p_i}$ is the center-of-mass (CoM) momentum of the system. In shell-model calculations, usually a one-body potential U is written into the Hamiltonian, which makes calculations more convenient,

$$\begin{aligned} H &= \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + U\right) + \sum_{i<j}\left(v_{ij} - U - \frac{p_i^2}{2Am} - \frac{\mathbf{p_i}\cdot\mathbf{p_j}}{Am}\right) \\ &= H_0 + V, \end{aligned} \tag{3}$$

with $H_0 = \sum_{i=1}^{A}(\frac{p_i^2}{2m} + U)$ having a one-body form. In the present calculations, U takes the WS potential of the ^{16}O core. V is the residual two-body interaction with corrections from the CoM motion.

The Hamiltonian is intrinsic, but it is difficult to write wave functions in a relative coordinate frame. As known, it is unfeasible to antisymmetrize the wave functions of $A > 7$ systems in a relative coordinates (e.g., the Jacobi coordinates). If wave functions are expressed in the laboratory coordinates, one should consider effects from the CoM motion which can cause spurious excitations. In the standard SM with the HO basis, the CoM spuriousity can be removed by the Lawson method [23]. Unfortunately, the Lawson method cannot be used within the Berggren complex-k basis due to the fact that the $\mathbf{R^2}$ matrix elements ($\mathbf{R}$ is the CoM coordinate) cannot be regularized using the complex scaling technique. By taking the coordinates of valence particles with respect to the core CoM, the cluster-orbital shell model (COSM) can remove the CoM excitation in the GSM calculations [24,25]. In the COSM coordinates, the translation invariance is preserved. However, the transformation of the realistic force to the COSM framework is very difficult to handle, as mentioned in Ref. [10]. In the present work, we calculate low excited states with excitation energies lower than 7 MeV, while the lowest CoM excitation energy is significantly larger than 7 MeV. Therefore, we have assumed that the CoM motion is in the s-wave and its effects on low-lying states calculated with the intrinsic Hamiltonian should not be remarkable.

We derive the effective two-body interaction from the CD-Bonn potential [26] which is defined in the relative momentum space. The bare interaction has a strong short-range repulsive core which comes from the high-momentum components of the interaction. To speed up the convergences of many-body calculations, usually the bare force is softened (renormalized). We soften the CD-Bonn interaction by using the $V_{\text{low-}k}$ method [27] in which high-momentum components above a certain cutoff Λ are integrated out.