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Hairs of discrete symmetries and gravity

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ABSTRACT

Gauge symmetries are known to be respected by gravity because gauge charges carry flux lines, but global charges do not carry flux lines and are not conserved by gravitational interaction. For discrete symmetries, they are spontaneously broken in the Universe, forming domain walls. Since the realization of discrete symmetries in the Universe must involve the vacuum expectation values of Higgs fields, a string-like configuration (hair) at the intersection of domain walls in the Higgs vacua can be realized. Therefore, we argue that discrete charges are also respected by gravity.

1. Introduction

It has been known for a long time that discrete sub-groups of gauge groups, the so-called discrete gauge symmetries, are not broken by gravitational interactions [1,2]. Effects of quantum gravity are studied by looking at various topologies of the metric tensor $g_{\mu\nu}$. If some gauge charges are separated from our Universe by metric change, the separated gauge charges cannot be completely hidden from our Universe because they leave long range flux lines. On the other hand, if global charges are separated from our Universe, the lost charges leave no hint to an observer in our Universe and he notices that global charges are not conserved in our Universe. Thus, gauge symmetries are not broken but global symmetries are broken by metric changes. This is the basic reasoning that discrete gauge symmetries are used in particle physics [3]. This top–down approach on discrete symmetries fits to the string compactification [4,5] because string theory does not allow any global symmetry.

In the bottom–up approach, the flux line argument is not so clear. It uses just the classical gauge fields and does not rely on the renormalizability in the theory of elementary particles. To be specific, let us consider a continuous symmetry U(1). If U(1) is a gauge symmetry, it should not have any gauge anomaly. If U(1) is a global symmetry, it may have a gauge anomaly U(1)–$G-G$ where G is a gauge group as in the Peccei–Quinn (PQ) global symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$ [6]. Obstructing the PQ symmetry needed for an "invisible" axion was based on this argument [7].

However, the absence of any gauge anomaly is not a guarantee for a gauged U(1) symmetry. Some global U(1) symmetries may not have any gauge anomaly. The difference in the gauge and global symmetries resides in the property on the local transformation, *i.e.* using a covariant derivative $D_\mu = \partial_\mu - iA_\mu$ in gauge theories, or just an ordinary one ∂_μ in global symmetries. A discrete subgroup of U(1) cannot know whether the mother U(1) is gauged or not. In the bottom–up approach, there must be some other reason for the effects of the metric change.

In this paper, we adopt the concept of "hair" which means that hair's thickness is the same at any distance from the surface of the head. At the surface, there must be fields at the surface for a hair to be defined. This definition excludes any possibility for hairs of global symmetries. In gauge theories, there are gauge fields at the surface. In gauge theories, the relation of the fields at the surface with the charge Q in the volume enclosed by the surface is provided by the equations of motion and current conservation. Existence of hairs is crucial in guaranteeing the symmetry in the presence of the gravitational interaction. It is known that black holes have gauge-charge hairs, which will be briefly commented in parallel with our method.

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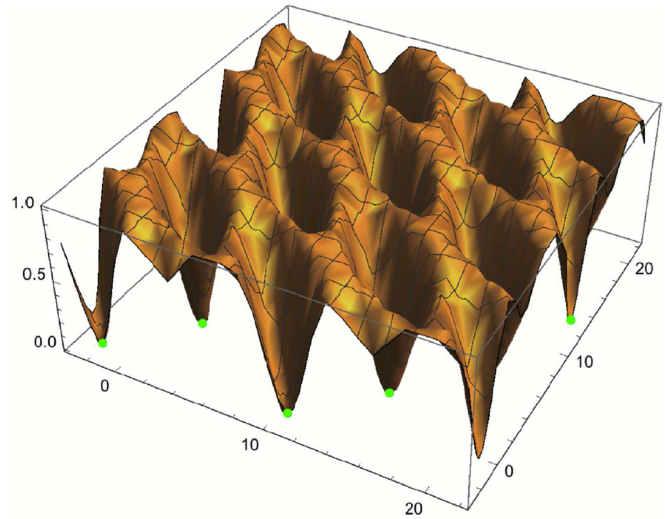


Fig. 1. Multiple discrete vacua. Some of minima are shown as green bullets. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

For a gauge charge Q, we have gauge fields spreading out from Q. Consider the current j^μ and the corresponding electric field $\mathbf{E}$ along a line to be interpreted as a hair. We can perform local transformations such that $\mathbf{E}$ is the same along a line but zero outside the line, which behaves as a hair.[1]

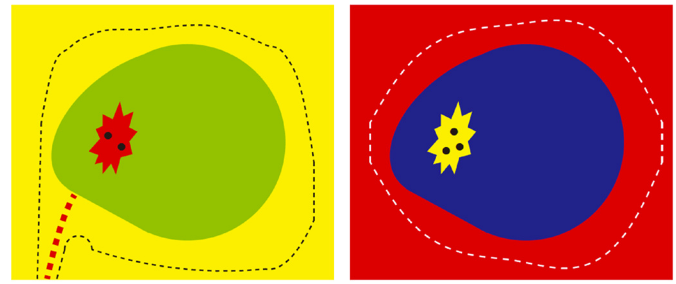
In this paper, we show how discrete charges can have hairs in the bottom–up approach, and derive that discrete symmetries are not broken by gravity. For an explicit presentation, we will present examples with the Abelian discrete symmetry $\mathbf{Z}_N$ and in particular with $\mathbf{Z}_2$ illustrations.

2. Discrete charges of $\mathbf{Z}_N$ vacua

A discrete symmetry is defined by the number of minima of the potential V such as in Fig. 1. Let us consider one minimum, say a green bullet in Fig. 1. We can choose the value of the Higgs field to be zero at that point so that the discrete symmetry is realized by the Wigner–Weyl manner. If it has a flat direction there, then one must consider a continuous symmetry, which has been spontaneously broken already. Not considering continuous symmetries, with the multiple vacua of Fig. 1, the discrete symmetry is good at any point of the minima. We will consider the discrete charges at such a minimum.

Realization of discrete symmetries in the Universe leads to domain walls [8]. In the "invisible" axion case [9], the Peccei–Quinn symmetry leads to $\mathbf{Z}_N$ domain walls [10]. For the Kim–Shifman–Vainstein–Zakharov "invisible" axion where there is only one vacuum [11], even the $\mathbf{Z}_1$ domain wall can be considered in the Universe evolution [12]. In this case, however, all space points except at the wall are in the same vacuum. Different vacua arise for the cases of $N \geq 2$. Two kinds of walled vacua are possible for $\mathbf{Z}_2$, viz. Fig. 2. Two vacua of $\mathbf{Z}_2$ are defined with discrete charges $q = 2n$ and $2n+1$, mod. 2 ($n =$ integer).

In Fig. 2(a), the (red, $Q_{\rm total} = 1$) vacuum is seen from the $q = 0$ (yellow) vacuum. A closed domain wall separates these two. This wall viewed from the yellow vacuum is symbolized by the limegreen color. In Fig. 2(b), the $q = 0$ (yellow, $Q_{\rm total} = 0$) vacuum is seen from the $q = 1$ (red) vacuum. The wall viewed from the red vacuum is symbolized by the blue color. In Fig. 2(a), the dashed boundary encloses the walled $q = 1$ vacuum. A scalar field ϕ in the $q = 0$ or $q = 1$ vacua is represented by $e^{iq\pi} R(\mathbf{x})$.



(a) (b)

Fig. 2. (a) A walled vacuum (red, $q_{\rm total} = 1$) seen from the $q = 0$ (yellow) vacuum. Inside the wall, the opposite $q = 1$ (red) vacuum is seen through a crack in the wall. This view of the wall is colored limegreen. (b) A walled vacuum (yellow, $q_{\rm total} = 0$) seen from the $q = 1$ (red) vacuum. This view of the wall is colored blue. Dashes represent closing surfaces and black dots represent particles. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Let us illustrate examples in $\mathbf{Z}_2$. Then, q can be 0 or 1. For $q = 0$, we use the field VEV $\phi = 0$.[2] For a ball of discrete charge q, the radius of the ball is determined by minimizing the energy

$$E_\omega = E + \omega \left[q - \frac{1}{2i} \int d^3x \, (\Phi^* \partial_t \Phi - \Phi \partial_t \Phi^*) \right], \tag{1}$$

where ω (with the energy dimension) is the Lagrange multiplier and q of the ball can be defined as

$$q = \frac{1}{2i} \int d^3x \, (\Phi^* \partial_t \Phi - \Phi \partial_t \Phi^*). \tag{2}$$

In the evolving Universe, the vacuum inside the ball expands such that ϕ in the red becomes constant. For a spherically symmetric $R(\mathbf{x})$, let us parametrize it as

$$\Phi = \sqrt{\frac{3k^3}{4\pi^4 \omega}}\, e^{i\omega t} \begin{cases} 1, & \text{for } 0 \leq r < \frac{\pi}{k} \\ 0, & \text{for } r > \frac{\pi}{k} \end{cases} \tag{3}$$

where π/k is the radius of the ball, and $\Phi = 0$ in the yellow part of Fig. 2(a). So, we obtain

$$\frac{1}{2i} \int_{\text{(inside dashed)}} d^3x \, (\Phi^* \partial_t \Phi - \Phi \partial_t \Phi^*) = 1.$$

The total charge q inside the dashed surface of Fig. 2(a) is 1, and the dashed string symbolizes this fact.

Definition of charge q by Eq. (2) is not by the Nöther current. It is simply defined by the vacuum expectation value (VEV) of the phase of a Higgs field Φ. To relate this charge q to the charge defined by the Nöther current, the t dependence of Φ is introduced as the example in Eq. (3). To make it an integer, the VEV which is designed as a constant[3] is appropriately chosen. Equation (1) is the matching condition to the charge q calculated by the Nöther current. Discrete symmetries in the Universe are realized by the VEVs of Higgs field Φ having degenerate minima as shown in Fig. 1. So, it is appropriate to figure out the discrete charges in the vacuum

[1] Here, the line is not a mathematical one but has some physical thickness. Thus, gauge charges can have hairs but global charges cannot, and metric changes know only hairs.

[2] If $\phi = -v$ corresponds to $q = 0$, we add a constant v to simplify the value of ϕ.

[3] In the connected portion in the Universe, the minimum of the potential with a fixed value of Φ is chosen everywhere.