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Investigation of classical radiation reaction with aligned crystals

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ABSTRACT

Classical radiation reaction is the effect of the electromagnetic field emitted by an accelerated electric charge on the motion of the charge itself. The self-consistent underlying classical equation of motion including radiation–reaction effects, the Landau–Lifshitz equation, has never been tested experimentally, in spite of the first theoretical treatments of radiation reaction having been developed more than a century ago. Here we show that classical radiation reaction effects, in particular those due to the near electromagnetic field, as predicted by the Landau–Lifshitz equation, can be measured in principle using presently available facilities, in the energy emission spectrum of 30-GeV electrons crossing a 0.55-mm thick diamond crystal in the axial channeling regime. Our theoretical results indicate the feasibility of the suggested setup, e.g., at the CERN Secondary Beam Areas (SBA) beamlines.

1. Introduction

The Lorentz equation is one of the cornerstones of classical electrodynamics and it describes the motion of an electric charge, an electron for definiteness (charge $e < 0$ and mass m), in the presence of an external, given electromagnetic field [1]. The Lorentz equation, however, does not take into account that, as the electron is being accelerated by the external field, it emits electromagnetic radiation, which in turn alters the trajectory of the electron itself (radiation reaction (RR)). The search for the equation of motion of an electron moving in a given external electromagnetic field, including self-consistently the effects of RR, has already been pursued since the beginning of the 20th century. By starting from the Lorentz equation of an electron in the presence of an external electromagnetic field and of the electromagnetic field produced by the electron itself, the so-called Lorentz–Abraham–Dirac (LAD) equation has been derived [2–4,1,5–8]. After mass renormalization RR effects result in two force terms in the LAD equation, one proportional to the Liénard formula for the radiated power and accounting for the energy–momentum loss of the electron due to radiation, the "damping term", and the other one, the "Schott" term, related to the electron's near field [8] and accounting for the work done by the field emitted by the electron on the electron itself [9]. Unlike the damping term, the Schott term, being proportional to the time derivative of the acceleration of the electron, 1) renders the LAD equation a non-Newtonian, third-order time differential equation; and 2) allows for unphysical features of the LAD equation as the existence of "runaway solutions", with the electron acceleration exponentially diverging in the remote future, even if, for example, the external field identically vanishes [1,5–11].

The origin of the inconsistencies of the LAD equation has been identified in [5]. The conclusion is that in the realm of classical electrodynamics, i.e., when quantum effects can be neglected, a "reduction of order" can be consistently carried out in the LAD equation, resulting in a second-order differential equation, known as the Landau–Lifshitz (LL) equation. Moreover, quoting Spohn [12], the physical solutions of the LAD equation "are on the critical manifold and are governed there by an effective second-order equation" which is the LL equation. Finally, the LL equation has been also derived from quantum electrodynamics in [13] (see also [14]).

The rapid progress of laser technology has renewed the interest in the problem of RR as the strong electromagnetic fields produced by lasers can violently accelerate the electron and consequently prime a substantial emission of electromagnetic radiation. Correspondingly, a large number of setups and schemes have been recently proposed to measure classical RR effects in electron-laser interaction [15–20] (we refer to the review [10] for previous proposals). However, experimental challenges either in the detection of relatively small RR effects or in the availability of sufficiently strong lasers has prevented so far any experimental test of the LL equation. Moreover, since RR effects are larger for ultrarelativistic electrons, reported laser-based experimental tests of the LL equation turn out to be sensitive mainly to the damping term in the LL

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equation, which has the most favorable dependence on the electron Lorentz factor.

In the present Letter we adopt a different perspective and put forward a presently feasible experimental setup to measure classical RR effects on the radiation field, generated in the interaction of ultrarelativistic electrons with an aligned crystal. The experiment can already be performed at, e.g., the CERN Secondary Beam Areas (SBA) beamlines. In fact, in the proposed setup 30-GeV electrons impinge into a 0.55-mm thick diamond crystal and emit a significant amount of radiation due to axial channeling [21–24]. Our numerical simulations indicate that in this regime RR effects substantially alter the electromagnetic emission spectrum. Moreover, unlike experimental proposals employing lasers, the distinct structure of the electric field of the crystal at axial channeling renders the emission spectrum more sensitive to a term in the LL equation originating from the controversial Schott term in the LAD equation. As we will see below, this term depends in general on the space-time derivatives of the background field. This feature makes our setup prominent also with respect to synchrotron facilities where the electron dynamics is dominated by the damping term. We also mention that at an electron energy $\varepsilon_0 = 30$ GeV and for a typical synchrotron radius $R = 1$ km, the relative electron energy loss per turn is $\Delta\varepsilon/\varepsilon_0 = 8.9\times 10^{-5}\varepsilon_0[\text{GeV}]^3/R[\text{m}] = 2.4\times 10^{-3}$ [25], which would induce too small effects on the emitted radiation to be measured. In addition, in order for the synchrotron to operate during many turns, the electron energy loss has to be precisely compensated preventing again any possibility of "accumulating" and measuring RR effects on the emitted radiation.

2. The physical model

When a high-energy electron impinges onto a single crystal along a direction of high symmetry, its motion can become transversely bound and its dynamics determined by a coherent scattering in the collective, screened field of many atoms aligned along the direction of symmetry (axial channeling) [21–24]. In this regime the electron experiences an effective potential in the transverse directions (continuum potential), resulting from the average of the atomic potential along the direction of symmetry. For the sake of simplicity, in the present and in the next section we assume that the atomic potential is due to a single string. By indicating as $\boldsymbol{z}$ the direction corresponding to the symmetry axis of the crystal and by $\boldsymbol{\rho} = (x, y)$ the coordinates in the transverse plane, with the atomic string crossing this plane at $\boldsymbol{\rho} = \boldsymbol{0}$, the continuum potential $\Phi(\rho)$ depends only on the distance $\rho = |\boldsymbol{\rho}|$ and it can be approximated as [23]:

$$\Phi(\rho) = \Phi_0\left[\ln\left(1+\frac{1}{\varrho^2+\eta}\right) - \ln\left(1+\frac{1}{\varrho_c^2+\eta}\right)\right], \tag{1}$$

where $\boldsymbol{\varrho} = \boldsymbol{\rho}/a_s$ and $\varrho_c = \rho_c/a_s$. Here, the parameters Φ_0, ρ_c, η, and a_s depend on the crystal and $\rho \le \rho_c$. A convenient choice to investigate classical RR effects is diamond, with, e.g., the $\langle 111\rangle$ as symmetry axis and for which $\Phi_0 = 29$ V, $\rho_c = 0.765$ Å, $\eta = 0.025$, and $a_s = 0.326$ Å. In fact, the relatively low value of Φ_0 as compared to other crystals allows one to neglect quantum effects also at relatively high electron energies. The depth $\Phi_M = \Phi(0)$ of the potential in diamond is such that $U_M = U(0) = -103$ eV, where $U(\rho) = e\Phi(\rho)$ is the electron potential energy (units with $\hbar = c = 1$ and $\alpha = e^2 \approx 1/137$ are employed throughout).

In general, the channeling regime of interaction features ultrastrong electromagnetic fields, which can lead to substantial energy loss of the radiating electron. In order for quantum effects to be negligible, we require that $\chi = \gamma_0 E/E_{cr} \ll 1$ [23], where γ_0 is the initial Lorentz factor of the electron, E is a measure of the amplitude of the electric field $\boldsymbol{E}(\boldsymbol{\rho}) = -\nabla\Phi(\rho) = (2\Phi_0/a_s)\boldsymbol{\varrho}/[(\eta+\varrho^2+(\eta+\varrho^2)^2]$ in the crystal, and $E_{cr} = m^2/|e| = 1.3\times 10^{16}$ V/cm is the critical electric field of QED. By employing $E \sim \Phi_M/\rho_c$ as an estimate of the electric field amplitude E, it is $\chi = 1.5\times 10^{-5}\varepsilon_0[\text{GeV}]|U_M[\text{eV}]|/\rho_c[\text{Å}]$.

In the classical regime $\chi \ll 1$ the electron dynamics including RR effects is described by the LL equation [5]. The LL equation for an electron with arbitrary momentum $\boldsymbol{p}(t) = m\gamma(t)\boldsymbol{\beta}(t)$, with $\gamma(t) = \varepsilon(t)/m = 1/\sqrt{1-\boldsymbol{\beta}^2(t)}$ and $\boldsymbol{\beta}(t) = \dot{\boldsymbol{r}}(t) = d\boldsymbol{r}(t)/dt$, reads:

$$\frac{d\boldsymbol{p}}{dt} = e\boldsymbol{E} + \frac{2}{3}\frac{e^2}{m}\Big\{e\gamma(\boldsymbol{\beta}\cdot\nabla)\boldsymbol{E} + \frac{e^2}{m}(\boldsymbol{\beta}\cdot\boldsymbol{E})\boldsymbol{E} - \frac{e^2}{m}\gamma^2[\boldsymbol{E}^2 - (\boldsymbol{\beta}\cdot\boldsymbol{E})^2]\boldsymbol{\beta}\Big\}. \tag{2}$$

Here the first two terms of the RR force originate from the Schott term in the LAD equation whereas the last "damping" one corresponds to the Liénard formula. Unlike the first "derivative" term, however, the second term of the RR force is strictly related to the damping one as only their sum ensures that the on-shell condition $\varepsilon(t) = \sqrt{m^2+\boldsymbol{p}^2(t)}$ is preserved during the electron motion.

Now, we assume that the crystal extends from $z = 0$ to $z = L$ and that at the initial time $t = 0$, the electron's position and velocity are $\boldsymbol{r}_0 = (x_0, 0, 0)$, with $0 < x_0 \le \rho_c$, and $\boldsymbol{\beta}_0 = (0, 0, \beta_{z,0})$, respectively ($\varepsilon_0 = m\gamma_0 = m/\sqrt{1-\beta_{z,0}^2}$). With these initial conditions, due to the symmetry of the potential $\Phi(\rho)$, it is $y(t) = 0$ and $E_y(\boldsymbol{\rho}) = 0$ along the electron trajectory. Thus, Eq. (2) substantially simplifies and only the equation

$$\frac{d\beta_x}{dt} = -\left(\frac{F_x}{\varepsilon} + \frac{2}{3}\frac{e^2}{m^2}\frac{dF_x}{dx}\beta_x\right)(1-\beta_x^2), \tag{3}$$

for $\beta_x(t)$ is needed below, with $F_x(x) = |e|E_x(x, 0)$.

If one first neglects RR, the total energy $\varepsilon(t) + U(|x(t)|)$ is a constant of motion. In the ultrarelativistic regime $\gamma_0 \gg 1$ of interest here and for typical crystal parameters it results $|\beta_x(t)| \ll 1$, such that $\varepsilon(t) \approx \varepsilon_0[1+\beta_x^2(t)/2]$ (see, e.g., [21–23]). Indeed, energy conservation implies that $|\beta_x(t)| \le \sqrt{2|U_M - U(x_0)|/\varepsilon_0} \ll 1$ (recall that $|U(\rho)| \sim 100$ eV [22,23]). Finally, with the considered initial conditions, the quantity $\beta_x(t)$ is periodic in time, with period $T_0 = \sqrt{8\varepsilon_0}\int_0^{x_0} dx/\sqrt{|U(x) - U(x_0)|}$ and angular frequency $\omega_0 = 2\pi/T_0$ [22].

3. Analytical results

The considerations above based on the single-string approximation allow us to evaluate the effects of RR on the electron dynamics analytically. In fact, as it can be verified *a posteriori*, it is safe to assume that $|\beta_x(t)| \ll 1$ and that $\beta_z(t) \approx 1$ also including RR. Thus, by multiplying Eq. (2) by $p_x(t)$ and by neglecting corrections proportional to $\beta_x^2(t) \sim |U_M|/\varepsilon_0$, it is easy to prove that (see also [5])

$$\varepsilon(t) = \frac{\varepsilon_0}{1 + (2/3)\alpha(\gamma_0/m^3)\int_0^t dt' F_x^2(x(t'))}, \tag{4}$$

where the integral is performed along the electron trajectory. In order to get an analytical insight on the motion of the electron, we assume here that $|x(t)| \ll a_s\sqrt{\eta}$, such that $F_x(x) \approx F_0 x/a_s\sqrt{\eta}$ and $dF_x(x)/dx \approx F_0/a_s\sqrt{\eta}$, where $F_0 = |e|E_0 = 2|U_0|/a_s\sqrt{\eta}$, with $U_0 = e\Phi_0$ ($U_0 = -29$ eV for diamond). Equation (3) with $1 - \beta_x^2(t) \approx 1$ and Eq. (4) show that the electron dynamics along the x direction is characterized by three time scales: one, $T_0 \approx 2\pi/\sqrt{F_0/\sqrt{\eta}\varepsilon_0 a_s}$, proper of the Lorentz dynamics and two additional,

$$\tau_s = \frac{6}{\alpha}\frac{\eta}{\gamma_0}\left(\frac{E_{cr}}{E_0}\right)^2\left(\frac{a_s}{x_0}\right)^2\lambda_C, \qquad \tau_d = \frac{3}{\alpha}\sqrt{\eta}\frac{E_{cr}}{E_0}a_s \tag{5}$$