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Generalized uncertainty principle as a consequence of the effective field theory

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ABSTRACT

We will demonstrate that the generalized uncertainty principle exists because of the derivative expansion in the effective field theories. This is because in the framework of the effective field theories, the minimum measurable length scale has to be integrated away to obtain the low energy effective action. We will analyze the deformation of a massive free scalar field theory by the generalized uncertainty principle, and demonstrate that the minimum measurable length scale corresponds to a second more massive scale in the theory, which has been integrated away. We will also analyze CFT operators dual to this deformed scalar field theory, and observe that scaling of the new CFT operators indicates that they are dual to this more massive scale in the theory. We will use holographic renormalization to explicitly calculate the renormalized boundary action with counter terms for this scalar field theory deformed by generalized uncertainty principle, and show that the generalized uncertainty principle contributes to the matter conformal anomaly.

1. Introduction

It is a universal prediction of almost all approaches to quantum gravity, that there is a minimum measurable length scale and it is not possible to make measurements below that scale. In perturbative string theory, the string length scale acts as the minimum measurable length scale. This is because in perturbative string theory, the smallest probe that can be used for analyzing any region of spacetime is the string, and so, spacetime not be probed at length scales below string length scale [1]. The existence of a minimum length scale in loop quantum gravity turns the big bang into a big bounce [2]. The generalized uncertainty principle has also been obtained from quantum geometry [3]. This has been done by taking into account the existence of an upper bound on the acceleration of massive particles [4,5]. So, the generalized uncertainty principle can also be motivated from a deformation of the geometry of spacetime by a constraint on the maximal acceleration of massive particles. It may be noted that the deformation of spacetime has also been analyzed using conformal transformations [6]. The energy needed to probe spacetime at length scales smaller than Planck length is more than the energy required to form a black hole in that region of spacetime. So, spacetime cannot be probed below the Planck scale, as this will lead to the formation of a mini black holes, which will in turn restrict the measurement of any phenomena below the Planck scale. Thus, the existence of a minimum measurable length scale can also be inferred from black hole physics [7,8]. On the other hand, the existence of a minimum measurable length scale is not consistent with the usual Heisenberg uncertainty principle. This is because according to the usual Heisenberg uncertainty principle, the length can be measured to arbitrary accuracy if the momentum is not measured. To incorporate the existence of a minimum measurable length scale in the Heisenberg uncertainty principle, one needs to modify it to a generalized uncertainty principle. However, as the Heisenberg uncertainty principle is related to the Heisenberg algebra, the deformation of the Heisenberg uncertainty principle also deforms the Heisenberg algebra [9–15].

The deformed Heisenberg algebra in turn deforms the coordinate representation of the momentum operator [9–15]. This corrects all quantum mechanical systems, including the first quantized equations of a field theory [16]. In fact, a covariant version of this deformed algebra is used to deform the field theories [17], and

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this covariant deformation is consistent with the existence of a minimum measurable time [18], apart from being consistent with the existence of a minimum measurable length. The gauge theories corresponding to such a deformed field theory has also been studied [19–21]. In this paper, we will analyze some theoretical aspects of such a deformed field theory. We will also use the holographic principle to understand the boundary dual of such a deformed field theory. The holographic principle states that the gravitational degrees of freedom in a region are encoded in the boundary degrees of freedom of that region. One of the most successful realization of the holographic principle is the gauge/gravity duality also known as the *AdS*/*CFT* correspondence [22–24]. This duality relates type IIB string theory on $AdS_5 \times S^5$ to $\mathcal{N} = 4$ super-Yang–Mills theory on its conformal boundary. It may be noted that even though the full string theory on $AdS_5 \times S^5$ is not understood, this duality can be used to map the weakly coupled limit of string theory to the strongly coupled gauge theory [25]. In fact, it can also be used to map a strongly coupled limit of the string theory to the weakly coupled limit of the gauge theory [26]. Thus, this duality can be used for analyzing the strongly coupled limit of the gauge theory by analyzing weakly coupled limit of the string theory. Since the weak coupling limit of the string theory can be approximated by ten dimensional supergravity, this duality is usually used to map the ten dimensional supergravity on $AdS_5 \times S^5$ to $\mathcal{N} = 4$ super-Yang–Mills theory on its conformal boundary. Furthermore, to suppress the loop contributions of the ten dimensional supergravity one takes the large N limit of the gauge theory. It may be noted that the UV divergences of the correlation functions on the gauge theory side need to be renormalized. However, these UV divergences are related to the IR divergences on the gravitational side of the duality. The IR divergences on the gravitational side are the same as near-boundary effects, and so, they can be dealt with by using holographic renormalization [27–30]. This is because the cancellation of the UV divergences does not depend on the IR physics, and this in turn implies that the holographic renormalization should only depend on the near-boundary analysis.

It may also be noted that even though the *AdS*/*CFT* conjecture has been mostly used in the context of string theory, this conjecture is actually a more general conjecture. In fact, the *AdS*/*CFT* conjecture has also been used for analyzing Rehren duality also known as algebraic holography [31–59]. The Rehren duality establishes the correspondence between an ordinary scalar field theory on *AdS* and a suitable conformal field theory on its boundary. In Rehren duality a space like wedge in *AdS* is mapped to its intersection with the boundary [33]. This sets up a bijection between the set of all wedges in the bulk and the set of all double-cones on the boundary. In fact, this bijection maps spacelike related bulk wedges to spacelike related boundary double-cones. Now for a net of local algebras on the bulk the Rehren duality defines a net of local algebras on the boundary. This is done by identifying the algebra for a given boundary double-cone with the bulk wedge algebra which restricts to it. In fact, another approach that relates a ordinary scalar field theory in the bulk to the conformal field theory on its boundary is the boundary-limit holography [34]. Thus, the main idea behind *AdS*/*CFT* conjecture has wider applications than relating type IIB string theory on $AdS_5 \times S^5$ to the $\mathcal{N} = 4$ super-Yang–Mills theory on its boundary.

Using this as a motivation, we will analyze the boundary dual of a scalar field theory with higher derivative corrections in the bulk. Higher derivative corrections to the scalar field theory have been predicted from discrete spacetime [49], spontaneous symmetry breaking of Lorentz invariance in string field theory [36], spacetime foam models [37], spin-network in loop quantum gravity [38], non-commutative geometry [39], Horava–Lifshitz gravity [40], and the existence of minimum length [9]. In fact, the existence of the string length scale also produces higher derivative corrections to the low energy phenomena [10,11]. Aspects of higher derivative terms have been investigated in cosmological inflation in [12–41]. In fact, motivated by the existence of a minimum length in string theory, higher derivative corrections to the scalar field theory in *AdS*/*CFT* has been recently analyzed [42]. As we are analyzing low energy effective phenomena, these higher derivative corrections are also expected to occur due to the derivative expansion in the effective field theory [71–77]. In this paper, we will analyze a scalar field theory deformed by generalized uncertainty principle, and observe that it contains higher derivative corrections. We will also analyze the physical meaning of these higher derivative terms. It has been suggested that the high energy excitations in the bulk will correspond to the CFT operators scaling as $\Delta_1 \sim N^{2/3}$ in five dimensions, or $\Delta \sim N^{1/4}$ in ten dimensions [24]. We observe that the deformation of the scalar field theory by the generalized uncertainty principle in the bulk produces CFT operators with this scaling property on the boundary. This implies that the deformation produced by the generalized uncertainty principle actually correspond to high energy excitation in the bulk, as was expected from effective field theory.

2. Deformed field theory

In this section, we will analyze the deformation of a massive scalar field theory on *AdS* by the generalized uncertain principle [19–21]. Furthermore, it will be demonstrated that the higher derivative corrections obtained from the generalized uncertainty principle will be exactly the same as the correction generated from a derivative expansion in the light of effective field theories [71–77]. The existence of minimum measurable length causes the following deformation of the uncertainty principle, and for a simple one dimensional system it can be written as $\Delta x \Delta p = [1 + \beta(\Delta p)^2]/2$ [9–15]. Here $\beta = \beta_0 \ell_{Pl}^2$ and β_0 is a constant normally assumed to be of order one, and this corresponds to taking the Planck length $\ell_{Pl} \approx 10^{-35}$ m as the minimum length scale. However, it is possible to take the minimum measurable length scale as an intermediate length scale ℓ_{Inter}, which is between the Planck length scale and electroweak length scale. In this case, the constant β_0 will be given by $\beta_0 \approx \ell_{Inter}^2/\ell_{Pl}^2$ [9]. It may be noted that this will change the value of β, and as we will demonstrate that β acts as another mass scale in the theory, this will change the value of that mass scale. However, in this paper, we will fix the value of $\beta_0 \approx 1$ by taking the Planck scale as the minimum measurable length scale. This deforms the Heisenberg algebra, as the Heisenberg algebra is closely related to the Heisenberg uncertainty principle. The deformed Heisenberg algebra in any dimension can be written as

$$[x^i, p_j] = i[\delta^i_j + \beta p^2 \delta^i_j + 2\beta p^i p_j]. \tag{1}$$

The coordinate representation of the deformed momentum, which is consistent with this algebra is [19]

$$p_\mu = -i\partial_\mu(1 - \beta\partial^\nu\partial_\nu). \tag{2}$$

We will analyze such a deformation of a free scalar field theory on *AdS*. The *AdS* metric can be written as

$$ds^2 = G_{MN}dx^M dx^N = L^2 z^{-2}[dz^2 + \delta_{\mu\nu}dx^\mu dx^\nu]. \tag{3}$$

The Laplacian on *AdS* is given by

$$\Box = z^{d+1}\partial_z(z^{-d}\partial_z) + \Box_0, \tag{4}$$

where $\Box_0 = \delta^{ij}\partial_i\partial_j$. Thus, a covariant version of the deformed momentum on *AdS* can be written as [42]

$$p_M = -i\nabla_M(1 - \beta\Box). \tag{5}$$