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Two-measure approach to breaking scale-invariance in a standard-model extension

Eduardo I. Guendelman [a], Hitoshi Nishino [b,*], Subhash Rajpoot [b]

[a] Department of Physics, Ben Gurion University of the Negev, Beer Sheva 84105, Israel
[b] Department of Physics & Astronomy, California State University, 1250 Bellflower Boulevard, Long Beach, CA 90840, United States

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ABSTRACT

We introduce Weyl's scale-invariance as an additional *global* symmetry in the standard model of electroweak interactions. A natural consequence is the introduction of general relativity coupled to scalar fields *à la* Dirac, that includes the Higgs doublet and a singlet σ-field required for implementing global scale-invariance. We introduce a mechanism for 'spontaneous breaking' of scale-invariance by introducing a coupling of the σ-field to a new metric-independent measure Φ defined in terms of four scalars ϕ^i $(i = 1, 2, 3, 4)$. Global scale-invariance is regained by combining it with internal diffeomorphism of these four scalars. We show that once the global scale-invariance is broken, the phenomenon (a) generates Newton's gravitational constant G_N and (b) triggers spontaneous symmetry breaking in the normal manner resulting in masses for the conventional fermions and bosons. In the absence of fine-tuning the scale at which the scale-symmetry breaks can be of order Planck mass. If right-handed neutrinos are also introduced, their absence at present energy scales is attributed to their mass terms tied to the scale where scale-invariance breaks.

In this letter we consider extending the standard model with global scale-invariance [1,2], the doomed symmetry that gave birth to the gauge principle and ultimately paved the way for implementing gauge-invariance as we know and practice today. A glance at the elementary particle mass spectrum attests to the fact that scale-invariance is a badly-broken symmetry of Nature. As we shall show, in the absence of fine-tuning, the scale at which the scale-invariance symmetry breaks turns out to be of order (reduced) Planck mass $M_P \approx 2.4 \times 10^{18}$ GeV.

Implementing scale-invariance in the standard model had been previously considered [3–5]. The main result in [3] was the elimination of the Higgs boson from the standard model particle spectrum. In [4],[1] scale-invariance was explicitly broken by hand. The philosophy advocated in the present work is different in spirit. In the present model scale-invariance is broken 'spontaneously', *via* a mechanism present in Two-Measure Theories, where we introduce a new measure of integration that is independent of the metric and is defined instead in terms of four scalar measure fields [7]. Such a procedure has been applied to the breaking of global scale-invariance in [8]. The transformation of the measure fields in conformally-invariant formulations of modified measure string theories was studied [9]. Two-measure theory formulation was further generalized to curved backgrounds for extended objects in [10].

Also in the present case, the measure fields participate in the conformal transformations by undergoing an internal diffeomorphism. The equations of motion of these measure fields naturally leads to the 'spontaneous' breaking of *global* scale-invariance. We finalize by recapitulating the salient features of the spontaneously-broken scale-invariant model.

Under global scale-transformation, the fundamental metric tensor $g_{\mu\nu}$ transforms as

$$g_{\mu\nu} \rightarrow e^{2\Lambda} g_{\mu\nu} \ , \tag{1}$$

where Λ is the parameter of *global* scale-transformations. The four-dimensional volume element transforms as

$$d^4x \sqrt{-g} \rightarrow e^{4\Lambda} d^4x \sqrt{-g} \ . \tag{2}$$

Since the vierbein $e_\mu{}^m$ and its inverse $e_m{}^\mu$ satisfy $e_\mu{}^m e_{\nu m} = g_{\mu\nu}$ and $e_m{}^\mu e_{n\mu} = \eta_{mn}$ where $(\eta_{mn}) = \text{diag.}\ (1, -1, -1, -1)$ is the tangent space metric, it follows that the transformation properties of $e_\mu{}^m$ and its inverse $e_m{}^\mu$ under Weyl's symmetry are

$$e_\mu{}^m \rightarrow e^{\Lambda} e_\mu{}^m \ , \quad e_m{}^\mu \rightarrow e^{-\Lambda} e_m{}^\mu \ . \tag{3}$$

* Corresponding author.
E-mail addresses: guendel@bgu.ac.il (E.I. Guendelman), hnishino@csulb.edu (H. Nishino), rajpoot@csulb.edu (S. Rajpoot).

[1] It was the first time that scale-invariance was implemented in the standard model in [4]. For subsequent works, see [5,6].

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We extend the standard model of particle interactions to include Weyl's scale-invariance as a global symmetry. The electroweak symmetry $SU(2)\times U(1)$ is extended to

$$G = SU(2)\times U(1)\times \widetilde{U}(1)_{\rm global} \ , \tag{4}$$

where $\widetilde{U}(1)_{\rm global}$ represents the Abelian symmetry associated with *global* scale-invariance. The additional particles introduced is a real scalar field σ [11–14] that transforms as a singlet under G. The distinct feature of the new symmetry is that under it fields transform with a real phase (a scaling-factor), whereas under the $SU(2)\times U(1)$ symmetries fields transform with complex phases.

Under $\widetilde{U}(1)_{\rm global}$ a generic field in the action is taken to transform as $e^{w\Lambda}$ with a scaling-weight w. Thus under $G = SU(2)\times U(1)\times \widetilde{U}(1)_{\rm global}$ the transformation properties of the entire particle content of the extended model are the following: The e-family ($g = 1$),

$$\Psi^{1q}_L = \begin{pmatrix} u \\ d\end{pmatrix} \sim (2,\tfrac{1}{3},-\tfrac{3}{2}) \ ; \qquad \Psi^{1l}_L = \begin{pmatrix} \nu_e \\ e\end{pmatrix} \sim (2,-1,-\tfrac{3}{2}) \ ;$$
$$\Psi^{1q}_{1R} = u_R \sim (1,\tfrac{4}{3},-\tfrac{3}{2}) \ ; \qquad \Psi^{1q}_{2R} = d_R \sim (1,-\tfrac{2}{3},-\tfrac{3}{2}) \ ;$$
$$\Psi^{1l}_{2R} = e_R \sim (1,-2,-\tfrac{3}{2}) \ , \tag{5}$$

and similarly for the μ-family ($g = 2$) and the τ-family ($g = 3$). All of these fermions have the same scaling-weight $w = -3/2$. The scalar bosons comprising the Higgs doublet φ and the real scalar σ,

$$\varphi \sim (2,-1,-1) \ ; \quad \sigma \sim (1,0,-1) \ , \tag{6}$$

with the common scaling-weight $w = -1$. We introduce W_μ and B_μ as the gauge potentials respectively associated with the $SU(2)$ and $U(1)$ symmetries. All of these gauge bosons have *zero* scaling-weight $w = 0$: $W_\mu \to W_\mu$ and $B_\mu \to B_\mu$. We suppress the $SU(3)$ of strong interactions as neglecting it will not affect our results and conclusions.

The action I_0 of the model is [4]

$$\begin{aligned} I_0 \equiv \int d^4x\sqrt{-g}\Bigg[& -\tfrac{1}{4} g^{\mu\rho} g^{\nu\sigma} (W_{\mu\nu}W_{\rho\sigma} + B_{\mu\nu}B_{\rho\sigma}) \\ & + g^{\mu\nu}(D_\mu\varphi)^\dagger(D_\nu\varphi) + \tfrac{1}{2} g^{\mu\nu}(\partial_\mu\sigma)(\partial_\nu\sigma) \\ & + \sum_{\substack{f=q,l\\ g=1,2,3\\ i=1,2}} \left(\overline{\Psi}{}^{gf}_L e_m{}^\mu \gamma^m D_\mu \Psi^{gf}_L + \overline{\Psi}{}^{gf}_{iR} e_m{}^\mu \gamma^m D_\mu \Psi^{gf}_{iR}\right) \\ & + \sum_{\substack{f=q,l\\ g,g'=1,2,3\\ i=1,2}} \left(\mathbf{Y}^f_{gg'} \overline{\Psi}{}^{gf}_L \varphi \Psi^{gf}_{iR} + \mathbf{Y}'^f_{gg'} \overline{\Psi}{}^{gf}_L \widetilde{\varphi} \Psi^{g'f}_{iR}\right) + \text{h.c.} \\ & - \tfrac{1}{2}(\beta\varphi^\dagger\varphi + \zeta\sigma^2)R + V(\varphi,\sigma) \Bigg] \ , \end{aligned} \tag{7}$$

where $\widetilde{\varphi} \equiv i\sigma_2\varphi^*$, the indices (g, g') are for generations, the indices $f = (q, l)$ refer to (quark, lepton) fields, $\mathbf{Y}^f_{gg'}$ or $\mathbf{Y}'^f_{gg'}$ are quark, lepton Yukawa couplings that define the mass matrices after symmetry breaking, the index $i = 1, 2$ is needed for right-handed fermions, while β and ζ are dimensionless couplings. The various D's acting on the fields represent the covariant derivatives constructed in the usual manner using the principle of minimal substitution. Explicitly,

$$\begin{aligned} D_\mu\Psi^{gf}_L &= \left(\partial_\mu + ig\tau\cdot W_\mu + \tfrac{i}{2}g' Y^{gf}_L B_\mu - \tfrac{1}{2}\omega_\mu{}^{mn}\sigma_{mn}\right)\Psi^{gf}_L \ , \\ D_\mu\Psi^{gf}_{iR} &= \left(\partial_\mu + \tfrac{i}{2}g' Y^{gf}_{iR} B_\mu - \tfrac{1}{2}\omega_\mu{}^{mn}\sigma_{mn}\right)\Psi^{gf}_{iR} \ , \\ D_\mu\varphi &= \left(\partial_\mu + ig\tau\cdot W_\mu - \tfrac{i}{2}g' B_\mu\right)\varphi \ . \end{aligned} \tag{8}$$

The Y^{gf}_L's , Y^{gf}_{iR}'s represent the hypercharge quantum numbers (e.g., $f = q$, $g = 1$, $i = 1$, $Y^{1q}_L = 1/3$, $Y^{1q}_{1R} = 4/3$, *etc.*), g and g' are the respective gauge couplings of $SU(2)$ and $U(1)$, while, $W_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths associated with $SU(2)$ and $U(1)$.

The spin connection $\omega_\mu{}^{mn}$ [15] is defined in terms of the vierbein $e_\mu{}^m$

$$\omega_{mrs} \equiv \tfrac{1}{2}(C_{mrs} - C_{msr} + C_{srm}) \ , \quad C_{\mu\nu}{}^r \equiv \partial_\mu e_\nu{}^r - \partial_\nu e_\mu{}^r \ , \tag{9}$$

while the affine connection $\Gamma^\alpha{}_{\mu\nu}$ is defined by

$$\Gamma^\rho{}_{\mu\nu} = \tfrac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right) \ . \tag{10}$$

The Riemann curvature tensor $R^\rho{}_{\sigma\mu\nu}$ is

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho{}_{\nu\sigma} - \partial_\nu\Gamma^\rho{}_{\mu\sigma} - \Gamma^\lambda{}_{\mu\sigma}\Gamma^\rho{}_{\nu\lambda} + \Gamma^\lambda{}_{\nu\sigma}\Gamma^\rho{}_{\mu\lambda} \ , \tag{11}$$

where $\Gamma^\rho{}_{\mu\nu}$, $R^\rho{}_{\sigma\mu\nu}$ and the Ricci tensor $R^\rho{}_{\mu\rho\nu} = R_{\mu\nu}$ have scaling-weight $w = 0$, while the scalar curvature $R = g^{\mu\nu}R_{\mu\nu}$ transforms with scaling-weight $w = -2$. The potential $V(\varphi,\sigma)$ is given by

$$V(\varphi,\sigma) = \lambda\,(\varphi^\dagger\varphi)^2 - \mu\,(\varphi^\dagger\varphi)\,\sigma^2 + \xi\,\sigma^4 \ , \tag{12}$$

where λ, μ, ξ are dimensionless couplings. Note that the scalar potential in this model consists of quartic terms only as required by Weyl's scale-invariance. The desired descent, a two-stage process, of G to $U(1)_{\rm em}$

$$G = SU(2)\times U(1)\times\widetilde{U}(1)_{\rm global} \xrightarrow{\langle\sigma\rangle} SU(2)\times U(1) \xrightarrow{\langle\varphi\rangle} U(1)_{\rm em} \ . \tag{13}$$

In the primary stage of symmetry breaking, scale-invariance will be broken. One straightforward method is to break the scale-symmetry by hand [4] by freezing the singlet-field by $\sigma = \Delta$. However, there is a more appealing field-theoretic way to break the scale-symmetry 'spontaneously'.

Implementing 'spontaneous' breaking of scale-invariance is going to be achieved by introducing an additional term to the action, that although scale invariant, can induce the spontaneous breaking of scale-invariance as we will show. The new term is a coupling of the σ-field to a metric-independent measure Φ defined in terms of four scalar fields ϕ^i $(i = 1, 2, 3, 4)$ as

$$\Phi = \varepsilon_{ijk\ell}\,\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu\phi^i)(\partial_\nu\phi^j)(\partial_\rho\phi^k)(\partial_\sigma\phi^\ell) \ . \tag{14}$$

In eq. (14), $\varepsilon_{ijk\ell}$ and $\varepsilon^{\mu\nu\rho\sigma}$ represent permutation symbols in internal-space and coordinate-space, the former has the same values in any coordinate frame.

Modified measures theory uses many types of measures of integration in the action. We could use for example the standard Riemannian integration measure $\sqrt{-g}$ and the above metric-independent measure Φ as

$$S = \int d^4x\,\Phi L_1 + \int d^4x\sqrt{-g}L_0 \ . \tag{15}$$

In our case, the second part has been already defined by (7), while L_1 is taken to be the following,

$$L_1 = K\sigma^n \quad (n \neq 0) \ . \tag{16}$$