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Charged-lepton decays from soft flavour violation

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ABSTRACT

We consider a two-Higgs-doublet extension of the Standard Model, with three right-handed neutrino singlets and the seesaw mechanism, wherein all the Yukawa-coupling matrices are lepton flavour-diagonal and lepton flavour violation is *soft*, originating solely in the non-flavour-diagonal Majorana mass matrix of the right-handed neutrinos. We consider the limit $m_R \to \infty$ of this model, where m_R is the seesaw scale. We demonstrate that there is a region in parameter space where the branching ratios of all five charged-lepton decays $\ell_1^- \to \ell_2^- \ell_3^+ \ell_3^-$ are close to their experimental upper bounds, while the radiative decays $\ell_1^- \to \ell_2^- \gamma$ are invisible because their branching ratios are suppressed by m_R^{-4}. We also consider the anomalous magnetic moment of the muon and show that in our model the contributions from the extra scalars, both charged and neutral, can remove the discrepancy between its experimental and theoretical values.

1. Introduction

In this paper we resume an old idea of two of us [1]: in a multi-Higgs-doublet model furnished with three right-handed neutrino singlets and the seesaw mechanism [2], lepton flavour may be conserved in the Yukawa couplings of all the Higgs doublets and violated solely in the Majorana mass terms of the right-handed neutrinos $\nu_{\ell R}$ ($\ell = e, \mu, \tau$), *viz.* in

$$\mathcal{L}_{\nu_R\,\mathrm{mass}} = -\frac{1}{2} \sum_{\ell_1,\ell_2} \bar{\nu}_{\ell_1 R} \left(M_R\right)_{\ell_1 \ell_2} C\, \bar{\nu}_{\ell_2 R}^T + \mathrm{H.c.}, \tag{1}$$

where C is the charge-conjugation matrix in Dirac space and M_R is a non-singular symmetric matrix in flavour space. Since $\mathcal{L}_{\nu_R\,\mathrm{mass}}$ has dimension three, the violation of the individual lepton flavour numbers L_ℓ and of the total lepton number $L = L_e + L_\mu + L_\tau$ is *soft*. Thus, in our framework $\mathcal{L}_{\nu_R\,\mathrm{mass}}$ is responsible for

1. the smallness of the light-neutrino masses,
2. lepton mixing,
3. violation of L, and
4. violation of L_e, L_μ, and L_τ.

In this context, lepton flavour-violating processes were explicitly investigated at one-loop order in ref. [3] and the following property of our framework was discovered. Let m_R denote the seesaw scale — the scale of the square roots of the eigenvalues of $M_R M_R^*$ — and n denote the number of Higgs doublets; it was found in ref. [3] that

i. the amplitudes of the lepton flavour-violating processes involving gauge bosons, like $\mu^- \to e^- \gamma$ and $Z^0 \to e^- \mu^+$, scale down as $1/m_R^2$ when $m_R \to \infty$; this holds even when in those processes the gauge bosons γ and Z^0 are virtual, *i.e.* they are off-mass shell;
ii. the amplitudes of the box diagrams for lepton flavour-violating processes like $\tau^- \to \mu^- \mu^- e^+$ and $\tau^- \to e^- e^- \mu^+$ also scale down as $1/m_R^2$ for a large seesaw scale;
iii. however, if $n \geq 2$, the amplitudes for lepton flavour-violating processes $\ell_1^- \to \ell_2^- \left(S_b^0\right)^*$, where $\left(S_b^0\right)^*$ is a virtual (off-mass shell) neutral scalar, approach a nonzero limit when $m_R \to \infty$. The non-decoupling of the seesaw scale in $\ell_1^- \to \ell_2^- \left(S_b^0\right)^*$ is an effect of the one-loop diagrams with neutrinos and charged scalars in the loop.

As a consequence, in our framework the amplitude of the process $\mu^- \to e^- e^+ e^-$, which derives from $\mu^- \to e^- \left(S_b^0\right)^*$ followed by $\left(S_b^0\right)^* \to e^+ e^-$, is *unsuppressed* in the limit $m_R \to \infty$. The same happens to the amplitudes of the four τ^- decays of the same type.

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Table 1
The experimental bounds on the branching ratios of some lepton flavour-changing decays. All the bounds are at the 90% CL. The first bound is from ref. [5], all the other bounds are from ref. [6].

$$\begin{aligned}
\mathrm{BR}\left(\mu^+ \to e^+\gamma\right) &< 4.2 \times 10^{-13} \\
\mathrm{BR}\left(\tau^- \to e^-\gamma\right) &< 3.3 \times 10^{-8} \\
\mathrm{BR}\left(\tau^- \to \mu^-\gamma\right) &< 4.4 \times 10^{-8} \\
\mathrm{BR}\left(\mu^- \to e^-e^+e^-\right) &< 1.0 \times 10^{-12} \\
\mathrm{BR}\left(\tau^- \to e^-e^+e^-\right) &< 2.7 \times 10^{-8} \\
\mathrm{BR}\left(\tau^- \to e^-\mu^+\mu^-\right) &< 2.7 \times 10^{-8} \\
\mathrm{BR}\left(\tau^- \to \mu^-\mu^+\mu^-\right) &< 2.1 \times 10^{-8} \\
\mathrm{BR}\left(\tau^- \to \mu^-e^+e^-\right) &< 1.8 \times 10^{-8}
\end{aligned}$$

It is important to stress that in our model the amplitude for $\mu^- \to e^-e^+e^-$ is unsuppressed because of the penguin diagrams for neutral-scalar emission in the $\mu^- \to e^-$ conversion; indeed, the penguin diagrams for either γ or Z^0 emission vanish in the limit $m_R \to \infty$. Thus, our model for lepton-flavour violation differs from, for instance, the scotogenic model discussed in ref. [4], wherein it is precisely the γ and Z^0 penguins that are instrumental in $\mu^- \to e^-e^+e^-$ and in muon–electron conversion in nuclei.[1]

Let us estimate a lower bound on m_R by using the experimental bounds, given in Table 1,[2] on the radiative decays $\ell_1 \to \ell_2\gamma$. The amplitude for any such decay has the form

$$\mathcal{A}\left(\ell_1^\pm \to \ell_2^\pm\gamma\right) = e\, \varepsilon_\rho^*\, \bar{u}_2 \left(i\sigma^{\rho\lambda}q_\lambda\right)\left(A_L\gamma_L + A_R\gamma_R\right)u_1, \tag{2}$$

where ε_ρ is the polarisation vector of the photon, u_1 and u_2 are the spinors of $\ell_1^\pm$ and $\ell_2^\pm$, respectively, and γ_L and γ_R are the projectors of chirality. The decay rate is given, in the limit $m_{\ell_2} = 0$, by

$$\Gamma\left(\ell_1^\pm \to \ell_2^\pm\gamma\right) = \frac{\alpha m_{\ell_1}^3}{4}\left(|A_L|^2 + |A_R|^2\right). \tag{3}$$

Knowing that A_L and A_R are suppressed by m_R^{-2}, one may estimate, just on dimensional grounds, that

$$A_{L,R} \sim \frac{1}{16\pi^2}\,\frac{m_{\ell_1}}{m_R^2}. \tag{4}$$

Using the first two bounds of Table 1 together with the experimental values for the masses and widths of the μ and τ, one may then derive the lower bounds $m_R \gtrsim 50$ TeV from $\mu^+ \to e^+\gamma$ and $m_R \gtrsim 2$ TeV from $\tau^- \to e^-\gamma$.

Thus, in the framework of ref. [3], if we take $m_R \gtrsim 500$ TeV then the radiative decays $\ell_1 \to \ell_2\gamma$ are invisible in the foreseeable future. On the other hand, because of the nonzero limit of the amplitudes for $\ell_1 \to \ell_2\left(S_b^0\right)^*$, the charged-lepton decays $\ell_1 \to \ell_2\ell_3^+\ell_3^-$ are unsuppressed when $m_R \to \infty$. It is the purpose of this paper to investigate those decays numerically in the framework of ref. [3], *assuming m_R to be so large that the radiative charged-lepton decays are invisible*. Then, m_R is also much larger than the masses of the scalars in the model, which we assume to be in between one and a few TeV.

As a sideline, in this paper we also consider the contributions of both the neutral and charged scalars to the anomalous magnetic moment a_ℓ of the charged lepton ℓ, with particular emphasis on a_μ.

In order to keep the number of parameters of the model at a minimum, we restrict ourselves to just two Higgs doublets. Anticipating our results, we find that *all* five decays $\ell_1 \to \ell_2\ell_3^+\ell_3^-$ may well be just around the corner, while at the same time the contributions of the non-Standard Model (SM) scalars of the model can make up for the discrepancy $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$ of the anomalous magnetic moment of the muon.

This paper is organised as follows. In section 2 we recall some results of ref. [3]. We then specialise to the case of just two Higgs doublets in section 3. We present the formulas for the contribution of the non-SM scalars to a_ℓ in section 4. Section 5 is devoted to a numerical simulation. In section 6 we summarise and conclude.

2. The lepton flavour-violating decays $\ell_1^- \to \ell_2^-\ell_3^+\ell_3^-$

2.1. The effective lepton flavour-violating interaction

The framework of ref. [3] assumes an n-Higgs-doublet setup wherein the violation of the family lepton numbers L_ℓ is soft. The corresponding Yukawa Lagrangian has the form

$$\mathcal{L}_{\mathrm{Yukawa}} = -\sum_{k=1}^{n}\sum_{\ell=e,\mu,\tau}\left[\phi_k^\dagger\,\bar{\ell}_R\left(\Gamma_k\right)_{\ell\ell} + \tilde{\phi}_k^\dagger\,\bar{\nu}_{\ell R}\left(\Delta_k\right)_{\ell\ell}\right]D_{\ell L} + \mathrm{H.c.} \tag{5}$$

The basic assumption is

$$\text{the matrices } \Gamma_k \text{ and } \Delta_k \text{ are diagonal, } \forall\, k = 1,\dots,n, \tag{6}$$

as is already implicit in equation (5). In that equation, the Higgs doublets and the left-handed-lepton gauge doublets are given by

$$\phi_k = \begin{pmatrix}\varphi_k^+ \\ \varphi_k^0\end{pmatrix}, \quad \tilde{\phi}_k = \begin{pmatrix}\varphi_k^{0*} \\ -\varphi_k^-\end{pmatrix}, \quad \text{and} \quad D_{\ell L} = \begin{pmatrix}\nu_{\ell L} \\ \ell_L\end{pmatrix}, \tag{7}$$

respectively.

The scalar mass eigenfields S_a^+ and S_b^0 are related to the φ_k^+ and φ_k^0 by

$$\varphi_k^+ = \sum_{a=1}^{n} U_{ka}S_a^+ \quad \text{and} \quad \varphi_k^0 = \frac{1}{\sqrt{2}}\left(v_k + \sum_{b=1}^{2n} V_{kb}S_b^0\right), \tag{8}$$

respectively [9]. The vacuum expectation values (VEVs) are $v_k/\sqrt{2}$. The unitary $n \times n$ matrix U diagonalises the Hermitian mass matrix of the charged scalars. The $2n \times 2n$ real orthogonal matrix $\tilde{V}$, which diagonalises the mass matrix of neutral scalar fields, is written as [9]

$$\tilde{V} = \begin{pmatrix}\mathrm{Re}\,V \\ \mathrm{Im}\,V\end{pmatrix} \quad \text{with} \quad V \equiv \mathrm{Re}\,V + i\,\mathrm{Im}\,V. \tag{9}$$

The matrix V is $n \times 2n$. We number the scalar mass eigenfields in such a way that $S_1^\pm = G^\pm$ and $S_1^0 = G^0$ are the Goldstone bosons. If there is only one Higgs doublet, *i.e.* when $n = 1$, the matrix V is simply $V = (i,\ 1)$ in the phase convention where $v_1 > 0$, and S_2^0 is the Higgs field of the SM.

We define the diagonal matrices

$$M_D = \sum_{k=1}^{n}\frac{v_k}{\sqrt{2}}\Delta_k, \quad M_\ell = \sum_{k=1}^{n}\frac{v_k^*}{\sqrt{2}}\Gamma_k = \mathrm{diag}\left(m_e, m_\mu, m_\tau\right). \tag{10}$$

[1] In this paper we do not address muon–electron conversion in nuclei because in order to do it we would need to specify, through *additional assumptions*, the Yukawa couplings of the extra Higgs doublets to the quarks. This is so because in our model muon–electron conversion in nuclei occurs — in the limit $m_R \to \infty$ — through $\mu^- \to e^-\left(S_b^0\right)^*$ followed by the $\left(S_b^0\right)^*$ coupling to quarks.

[2] Two new experiments are planned in search for lepton flavour-violation at the Paul Scherrer Institute. The MEG II experiment [7] plans a sensitivity improvement of one order of magnitude for $\mu^+ \to e^+\gamma$. The *Mu3e* experiment [8], which is in the stage of construction, aims at a sensitivity for $\mathrm{BR}\left(\mu^+ \to e^+e^-e^+\right)$ of order 10^{-16}.