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Light-cone singularities and transverse-momentum-dependent factorization at twist-3

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ABSTRACT

We study transverse-momentum-dependent factorization at twist-3 for Drell–Yan processes. The factorization can be derived straightforwardly at leading order of α_s. But at this order we find that light-cone singularities already exist and effects of soft gluons are not correctly factorized. We regularize the singularities with gauge links off the light-cone and introduce a soft factor to factorize the effects of soft gluons. Interestingly, the soft factor must be included in the definition of subtracted TMD parton distributions to correctly factorize the effects of soft gluons. We derive the Collins–Soper equation for one of twist-3 TMD parton distributions. The equation can be useful for resummation of large logarithms terms appearing in the corresponding structure function in collinear factorization. However, the derived equation is nonhomogeneous. This will make the resummation complicated.

Differential cross-sections of hadron collisions with observed small transverse momenta in final states, like Drell–Yan processes with the lepton pair at low transverse momentum $q_\perp$, are sensitive to the transverse motion of partons inside hadrons. To consistently describe these processes in QCD, one needs to establish their Transverse-Momentum-Dependent (TMD) factorization, in which nonperturbative- and perturbative effects are systematically separated. Using the proven factorizations one can extract from experiment various TMD parton distributions which contain information about inner structure of hadrons.

With TMD factorization the perturbative effects due to large energy scale Q in processes can be safely calculated with perturbation theory, and the nonperturbative effects are described with TMD parton distributions defined with QCD operators. To perform such a factorization one expands differential cross-section in the inverse power of Q. At the leading power of $1/Q$ or twist-2, TMD factorization has been established for several processes like e^+e^--annihilations [1], Drell–Yan processes [2,3] and Semi-Inclusive Deeply Inelastic Scattering (SIDIS) [4,5]. But, TMD factorization at next-to-leading power or twist-3 has been not studied at the rigorous level as that of factorizations at twist-2, although TMD factorization at twist-3 for some observables can be derived in a straightforward way at leading order of α_s. Some results about this can be found, e.g., in [6–12]. In this letter, we take unpolarized Drell–Yan process as an example to examine TMD factorization at twist-3.

It is well-known that light-cone singularities appear in twist-2 TMD parton distributions, if they are defined with light-cone gauge links as used in the definitions of parton distribution functions in collinear factorizations. These singularities appear at the next-to-leading order of α_s and can be regularized by using gauge links off light-cone direction. In general one can expect that twist-3 TMD parton distributions defined with light-cone gauge links will also have light-cone singularities as shown in [13]. With explicit calculations we will show that light-cone singularities already appear at leading order of α_s in twist-3 TMD factorization. We regularize them with off light-cone gauge links. Our result shows that a soft factor is needed even at the leading order of α_s to correctly factorize soft-gluon contributions. However, unlike the case of TMD factorization at twist-2, the soft factor has to be implemented in twist-3 factorization in an unique way from our result.

The regularization of the light-cone singularities introduces a regulator-dependence in TMD parton distributions. The dependence is governed by Collins–Soper equations. It is interesting to note that this type of equations can be used to resum large log's of $q_\perp/Q$ in perturbative coefficient functions in collinear factorization, as shown in [1,2,4]. The behavior of all structure functions of unpolarized Drell–Yan processes in the limit $q_\perp \to 0$ has been studied in [14,15], where one finds that it can be difficult to resum large log's in one of the structure functions. It is this structure

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which is relevant to the twist-3 TMD factorization studied here. We will derive the Collins–Soper equation for the twist-3 TMD parton distribution involved here. Our result shows that the resummation can be difficult because the needed Collins–Soper equation is not homogeneous.

We consider the unpolarized Drell–Yan process:

$$h_A(P_A) + h_B(P_B) \to \gamma^*(q) + X \to \ell^-(k_1) + \ell^+(k_2) + X, \qquad (1)$$

where initial hadrons are unpolarized. We will use the light-cone coordinate system. In this system a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. We introduce two light-cone vectors: $n^\mu = (0, 1, 0, 0)$ and $l^\mu = (1, 0, 0, 0)$, and two transverse tensors $g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu l^\nu - n^\nu l^\mu$, and $\epsilon_\perp^{\mu\nu} = \epsilon^{\alpha\beta\mu\nu} l_\alpha n_\beta$. In this system, the momenta of initial hadrons are given as:

$$P_A^\mu \approx (P_A^+, 0, 0, 0), \quad P_B^\mu \approx (0, P_B^-, 0, 0). \qquad (2)$$

The angular distribution is characterized by four structure functions [16]:

$$\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha_{em}^2}{2(2\pi)^4 S^2 Q^2}\Big[W_T(1+\cos^2\theta) + W_\Delta \sin(2\theta)\cos\phi + W_L(1-\cos^2\theta) + W_{\Delta\Delta}\sin^2\theta\cos(2\phi)\Big] \qquad (3)$$

where Ω is the solid angle of the observed lepton in Collins–Soper frame. In collinear factorization one finds in the limit $q_\perp \to 0$ that the first two structure functions have power divergences, $Q^2/q_\perp^2$ and $Q/q_\perp$, respectively [14,15]. The structure function W_Δ is relevant to our study here. It is noted that the resummation of large log terms in the perturbative coefficient function of W_T has been studied extensively. But it is not clear how to resum the large log terms in W_Δ, W_L and $W_{\Delta\Delta}$, as discussed in [14,15].

The hadronic tensor for the process is defined as:

$$W^{\mu\nu} = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{iq\cdot x}\langle h_A(P_A)h_B(P_B)|\bar q(0)\gamma^\nu q(0)|X\rangle \times \langle X|\bar q(x)\gamma^\mu q(x)|h_B(P_B)h_A(P_A)\rangle \qquad (4)$$

where spins of initial hadrons are averaged. For brevity we will take the electric charge of quarks as 1 in this work. We are interested in the kinematical region of $q_\perp^2 \ll Q^2$. In this region the tensor can be decomposed as [14]

$$W^{\mu\nu} = -g_\perp^{\mu\nu} W_\perp + \frac{1}{q^-}\left(q_\perp^\mu l^\nu + q_\perp^\nu l^\mu\right) W_l + \frac{1}{q^+}\left(q_\perp^\mu n^\nu + q_\perp^\nu n^\mu\right) W_n + \cdots, \qquad (5)$$

where $\cdots$ stand for terms whose effects are power-suppressed. We assume that the polarization of the leptons in the final state is not observed. Then we need only to consider the symmetric part of $W^{\mu\nu}$. In the small $q_\perp$ region, the above structure functions are related to W_T and W_Δ as:

$$W_T = W_\perp, \qquad W_\Delta = \frac{q_\perp}{Q}(W_n - W_l). \qquad (6)$$

For $W_\perp$ one can derive its twist-2 factorization [2], while for $W_{l,n}$ TMD factorization at twist-3 is needed.

TMD parton distributions are defined as hadronic matrix elements of QCD operators. As mentioned before, we will use the gauge link off the light-cone. This will regularize light-cone singularities as shown later. We introduce:

$$\mathcal{L}_u(\xi) = P\exp\left(-ig_s\int_{-\infty}^{0} d\lambda u\cdot G(\lambda u + \xi)\right), \qquad (7)$$

with $u^\mu = (u^+, u^-, 0, 0)$ and $u^- \gg u^+$. The TMD quark distributions of h_A are defined with the quark density matrix

$$\mathcal{M}_{ij}(x, k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ix\xi^- P_A^+ - i\xi_\perp\cdot k_\perp} \times \langle h_A|\Big(\bar q(\xi)\mathcal{L}_u(\xi)\Big)_j\Big(\mathcal{L}_u^\dagger(0)q(0)\Big)_i|h_A\rangle\Big|_{\xi^+=0}, \qquad (8)$$

where i and j are color- and Dirac-spinor indices. Similarly one has the quark density matrix of h_B, where the gauge link $\mathcal{L}_v$ is along the direction $v^\mu = (v^+, v^-, 0, 0)$ and $v^+ \gg v^-$. We consider that the hadrons are unpolarized. In this letter, we work with Feynman gauges which is a non-singular gauge. In singular gauges transverse gauge links at $\xi^- = \infty$ should be added to make the density matrix gauge invariant [17,18]. In the framework of Soft-Collinear Effective Theory (SCET) such transverse gauge links are also needed for gauge invariance as shown in [19]. The parameterization of the density matrix have been studied in [9,10,20,21], where twist-2- and twist-3 TMD quark distributions are defined. The density matrix for the unpolarized hadron is parameterized with TMD parton distributions up to twist-3 as:

$$\mathcal{M}_{ij}(x, k_\perp) = \frac{1}{2N_c}\Bigg\{f_1(x, k_\perp)\gamma^- + h_1^\perp(x, k_\perp)\sigma^{\mu-}\frac{k_{\perp\mu}}{M_A} + \frac{M_A}{P_A^+}\Big[e(x, k_\perp) + \frac{1}{M_A}f^\perp(x, k_\perp)\gamma\cdot k_\perp + h(x, k_\perp)\sigma^{-+} - \frac{1}{M_A}g^\perp(x, k_\perp)\epsilon_\perp^{\mu\nu}\gamma_\mu\gamma_5 k_{\perp\nu}\Big]\Bigg\}_{ij} + \cdots, \qquad (9)$$

where $\cdots$ stand for those of higher twist. In Eq. (9) the first two terms are of twist-2. f_1, $f^\perp$ and $g^\perp$ are defined with chirality-even operators, while e, h and $h_1^\perp$ are defined with chirality-odd operators. In this letter we will only consider the contributions to $W^{\mu\nu}$ involving chirality-even operators. The contributions involving chirality-odd operators are irrelevant to the behavior the small-$q_\perp$ or to the large log's of $q_\perp/Q$ appearing in the collinear factorization. They will be studied in a separate work. The defined distributions depend not only on the momentum fraction x and the transverse momentum $k_\perp$, but also on the renormalization scale μ and the parameter ζ_u defined as $\zeta_u^2 = (2u\cdot P_A)^2/u^2$. Similarly one can define TMD antiquark distributions of h_B. The parameter ζ_v related to h_B is then defined as $\zeta_v^2 = (2v\cdot P_B)^2/v^2$. In Eq. (9) it is implied that ζ_u is large but finite and any contribution proportional to power of ζ_u^{-1} is neglected.

TMD factorization at leading order of α_s can be derived straightforwardly by the diagram expansion. The factorized form for W_l and W_n have been essentially derived in [9]. If we consider the case that the process is initiated by a quark from h_A and an antiquark from h_B, then one needs to consider up to twist-3 diagrams given in Fig. 1. In Fig. 1 the bubbles are jet-like Green functions related to the initial hadrons. From Fig. 1a one can obtain not only twist-2 contributions to $W_\perp$, but also twist-3 contributions to W_l and W_n. At the considered order of Q^{-1} the bubbles or jet-like Green functions in Fig. 1a are essentially the quark density matrices like the one defined in Eq. (8). We have:

$$W^{+\nu}\Big|_{1a} = W^{\nu+}\Big|_{1a} = \frac{q_\perp^\nu}{N_c P_B^-}\int d^2k_{A\perp}d^2k_{B\perp}\frac{\vec{k}_{B\perp}\cdot\vec{q}_\perp}{q_\perp^2} \times \delta^2(k_{A\perp} + k_{B\perp} - q_\perp) f_1(x, k_{A\perp})\bar f^\perp(y, k_{B\perp}), \qquad (10)$$