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Redundant and physical black hole parameters: Is there an independent physical dilaton charge?

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ABSTRACT

Black holes as solutions to gravity theories, are generically identified by a set of parameters. Some of these parameters are associated with black hole physical conserved charges, like ADM charges. There can also be some "redundant parameters." We propose necessary conditions for a parameter to be physical. The conditions are essentially integrability and non-triviality of the charge variations arising from "parametric variations," variation of the solution with respect to the chosen parameters. In addition, we prove that variation of the redundant parameters which do not meet our criteria do not appear in the first law of thermodynamics. As an interesting application, we show that dilaton moduli are redundant parameters for black hole solutions to Einstein–Maxwell–(Axion)–Dilaton theories, because variations in dilaton moduli would render entropy, mass, electric charges or angular momenta non-integrable. Our results are in contrast with modification of the first law due to scalar charges suggested in Gibbons–Kallosh–Kol paper [1] and its follow-ups. We also briefly discuss implications of our results for the attractor behavior of extremal black holes.

1. Introduction

Solutions to generic physical theories are specified by a set of parameters. These parameters are generically integrals of motion that are specified by initial and boundary conditions. Although there could be cases where some of these parameters take discrete-values, here we will only focus on real-valued parameters which can be varied continuously and define the "solution space" as the space of solutions spanned by these parameters.

All physical observables associated with a solution are functions of these parameters. In particular, among the physical observables there are conserved charges. The celebrated Noether theorem [2], while relating the conserved charges to symmetries of the theory, provides the functional form of conserved charges on this solution space.

In diffeomorphism invariant gravity theories, where we do not necessarily have (globally defined) time-like Killing vectors, the notion of "conservation" should be handled with special care. In addition, application of usual Noether theorem may face various challenges (e.g. see [3] and references therein for a review). To tackle these issues various different proposals and formulations have been proposed. However, there are aspects of this problem which still remain as a matter of debate to date.

As it may generically happen, physical observables may only be functions of a subset of parameters spanning the solution space. Or in other words, a part of the solution parameters may not appear in any physical observable. This can, in particular, happen in theories with local gauge symmetry (like diffeomorphisms in gravity) or in theories with field redefinition symmetry at the level of classical action. For example, some of the solution parameters could be gauge artifacts which may be removed in different coordinate systems or by a choice of gauge. It may also happen that the theory enjoys a field redefinition symmetry and some of the parameters may be related to a choice of dynamical fields to describe the system. An important example, which we consider and analyze here, is the shift symmetry in systems with a dilaton field. One would hence face the question which of the solution parameters are really physical ones.

An answer to this question, which is implicitly used in the literature, can be the following: any parameter that can be removed by a symmetry transformation (transformations which do not change the equations of motion and a given boundary condition) is redundant, while the parameters which appear explicitly in the conserved charges like mass, angular momentum etc. are physical. This inaccurate resolution has shortcomings in the following

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two situations. Firstly, it is now an established fact that there are some diffeomorphisms or gauge transformations to which one can associate non-trivial conserved charges [4]. In such cases, we are dealing with family of diffeomorphic, but still distinct, geometries [5–9] (or in generic case, gauge equivalent, but still distinct solutions, e.g. see [10]) which are specified by a number of arbitrary functions. These families of solutions may hence be labeled by infinite parameters which are e.g. related to the Fourier modes of these functions and there is a well-defined charge associated with each of these parameters. Secondly, there can be parameters removable by the action of a symmetry, while they appear explicitly in the well-established conserved charges like mass etc. An important example of such a parameter is the dilaton modulus in the dilaton shift symmetry.

While the method and algorithm we provide can be used in a wider context, here we would tackle the question described above for a specific class of solutions to gravity theories, the black holes. We mainly focus on the family of stationary black holes, those which have a time-like Killing vector outside their (event) horizon. It is now established that black holes generically Hawking radiate, a black body radiation emitted from any thermal system at the Hawking temperature and there should be an entropy associated with them [11,12]. It is also established that black holes obey laws of thermodynamics [13,14]. Black hole thermodynamical quantities, which are either the extensive conserved charges, or the intensive (chemical) potentials, are all functions over the black hole solution space. Our goal here is to provide unambiguous criteria and algorithm to distinguish the physical and redundant parameters of these solutions.

2. Physical vs. redundant solution parameters

Let us start by crystallizing the definition of solution parameters described in the introduction.

Definition 1. Given a Lagrangian density $\mathcal{L}$ and a solution to its e.o.m's with a given boundary condition, in some specific gauge and coordinate system, "solution parameters" p_i are constants in dynamical fields each of which can be varied while e.o.m's are still satisfied.

This definition makes a clear distinction between solution parameters and conserved charges attributed to a solution; solution parameters and conserved charges are conceptually different entities, which may or may not be related. To keep the distinction in mind, we denote the set of parameters by p_i, while the standard notation mass M, angular momentum J, electric charge Q, entropy S etc. is used for the conserved charges. Notice that this definition covers parameters which may appear in a solution and correspond to the "residual symmetries and charges" [7] discussed in the introduction. On the other hand, it excludes parameters of the theory, constants which appear explicitly in the Lagrangian, like the Newton constant G or the cosmological constant Λ. Moreover, the definition clarifies that these parameters cannot be constrained or related to each other through equations of motion (e.o.m) or boundary conditions.

2.1. Charges vs. charge variations and the integrability

Solution parameters can be related to the conserved charges of a solution through symmetries of the theory and/or solution. Conserved charges may be calculated by different methods. One can recognize two classes of such methods: those which provide a prescription to calculate the charges directly, and the methods which calculate charge *variations* first, and if integrable, then the finite charges. Noether charge [2,15], Komar charge [16], ADM method [17] and its later developments like Brown–York [18], and ADT methods [19], are examples in the first category (reviewed e.g. in [20]). In these methods, the conserved charges are read directly from the solution (or possibly its subtraction off a reference solution). As examples of methods in the second category, quasi-local method [21], covariant phase space formulation [3,14,22–28] and solution phase space method (SPSM) [29] can be mentioned.

The methods based on charge variations, especially in the context of gravity, are more precise, applicable to a wider range of solutions and may be uniformly applied to solutions with various asymptotic behavior. In these methods integrability of charge variations may yield non-trivial constraints on physical observables. The main idea we propose in this paper is to use the integrability conditions to distinguish between physical and redundant solution parameters. We employ SPSM which we find the more rigorous method among those in the charge variation class. Here, we briefly highlight the main ingredients and features of this method. For more details, the reader can refer to [29] or [30,31].

2.2. Solution Phase Space Method, a quick review

SPSM elaborates on the connection between solution parameters and conserved charges. To calculate a charge variation four inputs are needed: 1) A Lagrangian $\mathcal{L}$ on d dimensional spacetime with coordinates x^μ; 2) A symmetry to which the charge variation is attributed; 3) A solution to the e.o.m of the theory specified with dynamical field configuration Φ (e.g. metric $g_{\mu\nu}$, gauge field A_μ, scalar field ϕ, etc.); 4) A perturbation around the solution $\delta\Phi(x^\mu)$ satisfying linearized e.o.m.

Based on covariant phase space formulation [14,22,24,27,28], SPSM combines the four inputs in a simple relation, to introduce a charge variation δH_ϵ,

$$\delta H_\epsilon \equiv \int_\Sigma \boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi; \Phi) = \oint_{\partial\Sigma} \boldsymbol{k}_\epsilon(\delta\Phi; \Phi)\,, \tag{1}$$

where Σ is a (codimension one) Cauchy surface and $\partial\Sigma$ is its (codimension two) boundary. δH_ϵ is conserved if it is independent of the choice of Σ. The $d-1$-form $\boldsymbol{\omega}$ (called symplectic current) is on-shell closed ($d\boldsymbol{\omega}=0$ on-shell) and its form is determined through the Lagrangian $\mathcal{L}$, e.g. see [14,27,28]. To write the second equation we have used $\boldsymbol{\omega}=d\boldsymbol{k}$ on-shell and the Stokes' theorem. Explicit form of $\boldsymbol{k}$ for generic Lagrangians may be found e.g. in [31].

Information of the symmetry is in ϵ, which is in general a combination of a diffeomorphism generator vector field ξ^μ and a gauge transformation λ, denoted by $\epsilon=\{\xi^\mu,\lambda\}$ [29,32]. They act on fields as $\delta_\epsilon\Phi \equiv \mathcal{L}_\xi\Phi + \delta_\lambda A$ where $\mathcal{L}_\xi$ is Lie derivation and $\delta_\lambda A_\mu = \partial_\mu\lambda$.

In SPSM, ϵ is taken to be some specific subset of general diffeomorphisms and gauge transformations, called *symplectic symmetries*, for which

$$\boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi, \Phi) = 0\,, \tag{2}$$

over the specified set of solutions Φ and $\delta\Phi$. This nice feature makes the conservation to be guaranteed and renders the conserved charge variations to be independent of the codimension-2 integration surface $\partial\Sigma$. Therefore, the charges can be obtained from integrating $\boldsymbol{k}_\epsilon$ over any smooth and closed codimension-2 surface $\mathcal{S}$ inside the bulk which encompass any non-smoothness, singularity or closed-time-like-curves of the solution [29].

Among all solutions of the theory, SPSM focuses on those which may be denoted by $\Phi(x^\mu; p_i)$ where p_i are the parameters discussed in Definition 1. For the last but not least input, the standard condition which is imposed on $\delta\Phi$ is that it satisfies linearized